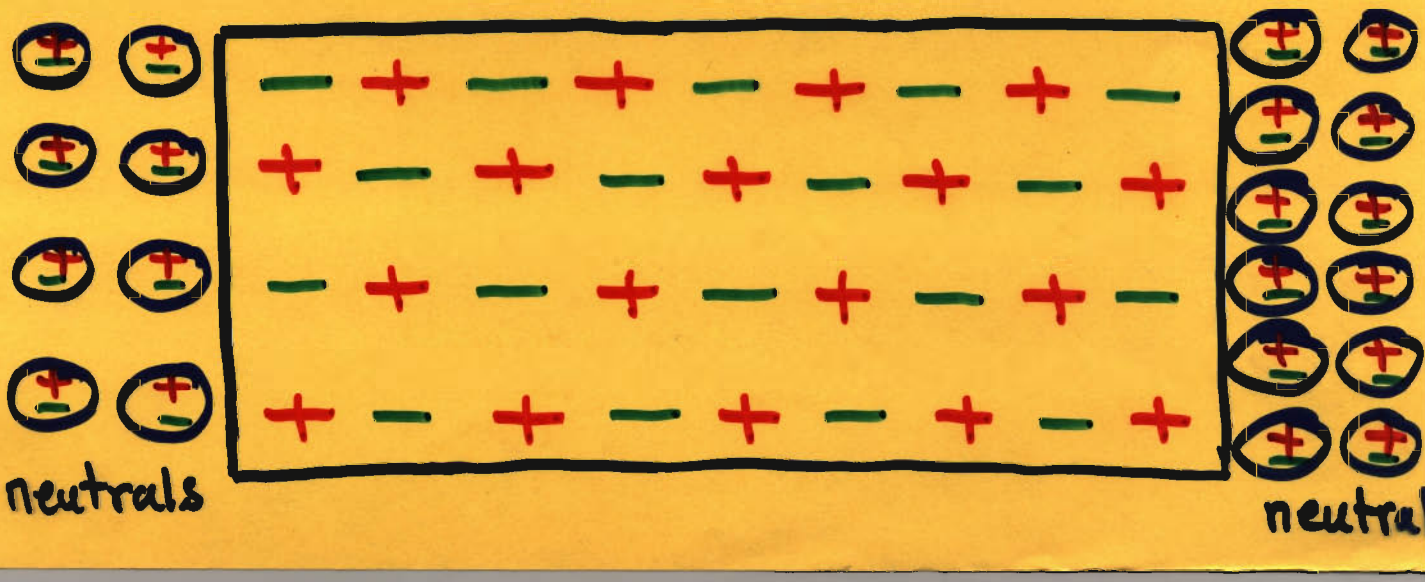
Basic Plasma Parameters and Particle Motion

PLASMA FREQUENCY

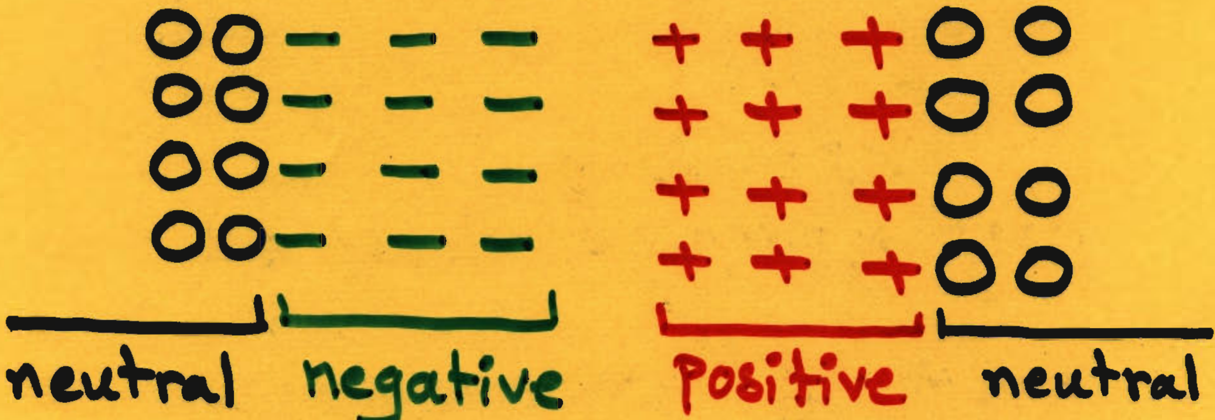
Physical Picture:

- Consider an ideal stiuation of an ionized gas. An assembly of electrons and ions, uniformly distributed.
- Bulk plasma is now neutral.
- Assume e^{-s} and i^{+s} are motionless i.e. no thermal motion.
- Let us distribute the picture



• Now, Let us disturb the plasma.

Allow some electrons to move into another region thus leaving positive charge.



• Result: (1) Local charge gives rise to an electric field $\bar{E}$.

(2) electrons respond more rapidly to $\bar{E}$

(3) electrons acquire kinetic energy.

(4) when electrons return to initial position, plasma becomes neutral and $\bar{E}$ goes back to zero.

(5) All of initial electrostatic potential energy "at purturbation" converted to kinetic energy.

(6) K.E. carries electrons past their initial position.

(7) Plasma, again, becomes non-neutral

(8) Electric field is set-up again and retards electron motion.

(9) electron velocity decreases and reaches x 0.

(10) K.E. Converted to electrostatic potential energy

(11) Situation is identical to that at initial perturbation with $\bar{E}$ opposit.

(12) Continue for a second half cycle, energy shifts from P.E. to K.E. and back again.

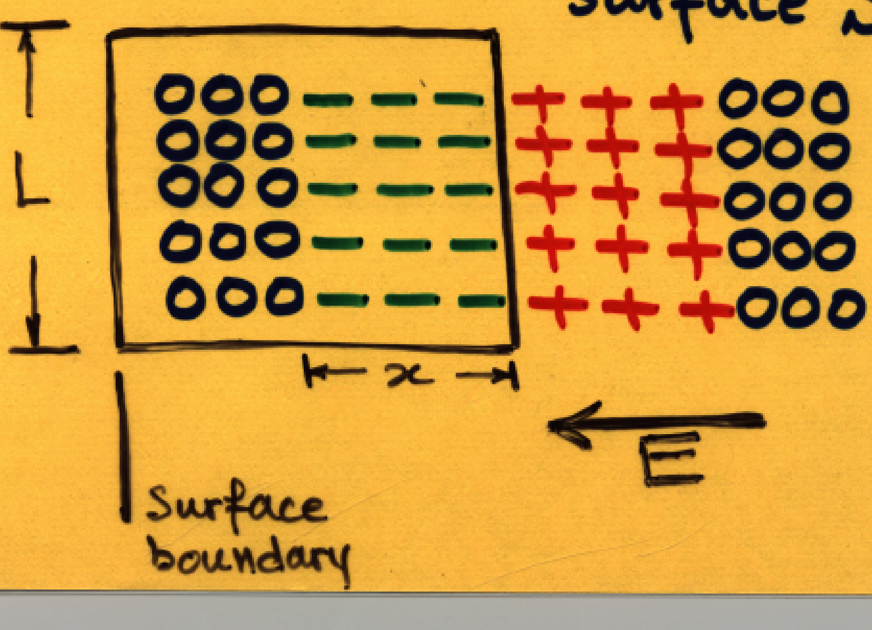
Mathematically:

equation of motion (single electron):

$$m\ddot{x} = -eE$$

Gauss Theorem: $$\int_S \bar{E} \cdot d\bar{S} = \frac{q}{\epsilon_0}$$

($q \rightarrow$ charge contained in closed surface S)



$$\int_S \bar{E} \cdot d\bar{S} = -LE$$

Let n_0 be equilibrium particle number density

$$q = -Lxn_0e$$

$$\therefore \int_S \bar{E}\cdot d\bar{s} = -LE = \frac{q}{\epsilon_0} = \frac{-Lxn_0e}{\epsilon_0}$$

$$\ddot{x} = -\frac{e}{m}E = -\frac{e}{m}\cdot\frac{xn_0e}{\epsilon_0}$$

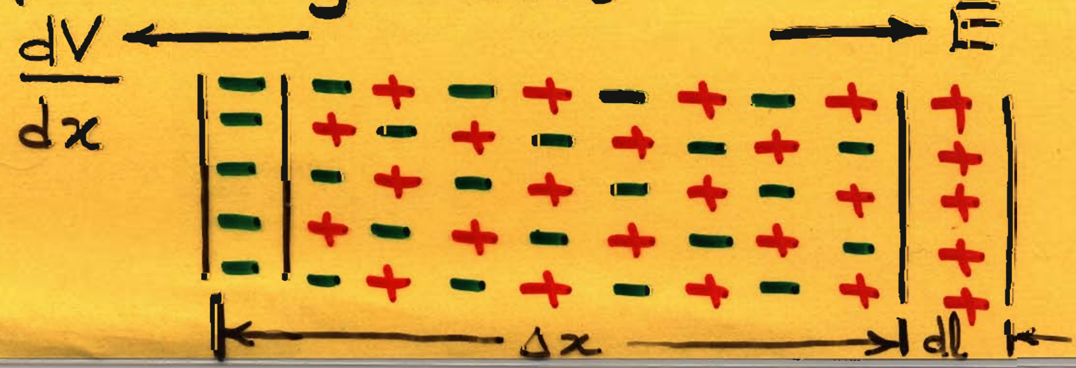
$$= -\underbrace{\frac{n_0e^2}{m\epsilon_0}}_{\omega_{pe}^2}\, x$$

$\omega_{pe}^2 \Longrightarrow$ electron plasma frequen

$$\therefore \ddot{x} + \omega_{pe}^2\, x = 0 \Rightarrow \text{harmonic motion}$$

numerical value: $\underset{(Hz)}{f_{pe}} \approx 9\sqrt{n_0}$ (m^{-3})

one can get the same answer when looking into the electric field as the potential gradients



upon perturbation, thin layer of thickness l develope with $\frac{dl}{dx} \ll 1$ and electron charge is "temporarly" in excess of the average.

Poisson's equation:

$$e\, dn_e = e n_e \frac{dl}{dx} = \epsilon_o \frac{d^2 V}{dx^2}$$

Force on particle of charge e and mass m

$$-e \frac{dV}{dx} = -\frac{e^2}{\epsilon_o} n_e l = m \frac{d^2 l}{dt^2}$$

$$m \frac{d^2 l}{dt^2} + \frac{e^2 n_e}{\epsilon_o} l = 0$$

Simple harmonic motion $l = l_o \cos(\omega t + \phi$

with $l_o \equiv$ amplitude and $\phi \equiv$ phase between l &

with $\omega \equiv$ angular frequency $\equiv \omega_{P_e} = \left(\frac{n_e e^2}{m \epsilon_o}\right.$

one can also get the same answer when looking to a plasma slab and balance the pressure, then utilizing Poisson's equation.

Electron plasma frequency for discharges is in the microwave region (1–10 GHz).

Importance of plasma frequency:

(1) Communication

$\omega < \omega_{pe}$ { – ground-to-ground via reflection from ionosphere

$\omega > \omega_{pe}$ { – ground-to-space through the ionosphere

$\omega = \omega_{pe}$ "cut-off" { – diagnostic tool to measure electron density

(2) Plasma heating in Fusion Devices ($\omega > \omega_{pe}$)

Similarly: an ion plasma frequency can be obtained

$$\omega_{p_i}^2 = \frac{n_i e^2}{m_i \epsilon_0}$$

with both electrons and ions oscillate it is "as if" ions oscillate in a "sea" of electrons.

The assumption made to solve for electron oscillation is based on ions being at rest "due to mass ratio", and the "Similarity" to obtain ion oscillations assumes perturbing "<u>only</u>" ions.

If these assumptions are to be re-visited, then the equation of motion when perturbing both electrons and ions would be:

$$\frac{d^2 l}{dt^2} + \underbrace{\frac{e^2 n_e}{\epsilon_o m_e}}_{\omega_{p_e}^2} l + \underbrace{\frac{e^2 n_i}{\epsilon_o m_i}}_{\omega_{p_i}^2} l = 0$$

$$\frac{d^2 l}{dt^2} + (\omega_{p_e}^2 + \omega_{p_i}^2)\, l = 0$$

Then the natural plasma frequency is:

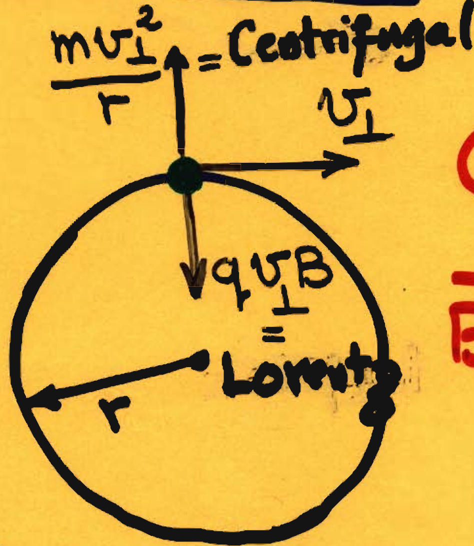
$$\omega_p = (\omega_{p_e}^2 + \omega_{p_i}^2)^{1/2}$$

for $m_i \gg m_e \longrightarrow \omega_p \approx \omega_{p_e}$

Plasma oscillations are damped by Collisions and also Collisionlessly (Landau damping)

Gyro "Cyclotron" Frequency:

a charged particle in a uniform magnetic field gyrates around the B-field in an orbit with radius r



$\odot\ \bar{B}$

$\bar{B} = \hat{z} B_o$

Equation of motion: $m\dfrac{d\bar{v}}{dt} = q(\bar{v} \times \bar{B})$

$$m\frac{dv_x}{dt} = q v_y B_o$$

$$m\frac{dv_y}{dt} = -q v_x B_o$$

$$m\frac{dv_z}{dt} = 0$$

$$m\frac{d^2 v_x}{dt^2} = q B_o \frac{dv_y}{dt}$$

$$= -\frac{q^2 B_o^2}{m} v_x$$

i.e. $$\frac{d^2 v_x}{dt^2} + \left(\frac{q^2 B_o^2}{m^2}\right) v_x = 0$$

This means that the charged particle is exercizing a harmonic motion and is orbiting with a frequency $= \frac{qB_o}{m} = \omega_c$

Then:

$$v_x = v_\perp \cos(\omega_c t + \phi) \quad \text{and}$$
$$v_y = -v_\perp \sin(\omega_c t + \phi)$$
$$v_z = v_{z_o}$$

Looking to the forces:

Lorentz force = Centrifugal force

$$q v_\perp B_o = \frac{m v_\perp^2}{r}$$

$$\therefore r = \frac{v_\perp}{\left(\frac{qB_o}{m}\right)} = \frac{v_\perp}{\omega_c}$$

gyroradius "or Larmor Radius" (r_L)

and:

$$x = r_L \sin(\omega_c t + \phi) + (x_o - r_L \sin\phi)$$
$$y = r_L \cos(\omega_c t + \phi) + (y_o - r_L \cos\phi)$$
$$z = z_o + v_{z_o} t$$

Electrons and ions will gyrate around B in opposite directions at different frequencies ($\omega_{ce} > \omega_{ci}$ >

$$\omega_{ce} = \frac{eB}{m_e}$$

$$\omega_{ci} = \frac{ZeB}{m_i}$$

and:

$$r_{Le} = \frac{v_e}{\omega_{ce}} = \frac{1}{B}\left(\frac{8 m_e T_e}{\pi e}\right)^{1/2}$$

$$r_{Li} = \frac{v_i}{\omega_{ci}} = \frac{1}{ZB}\left(\frac{8 m_p T_i A}{\pi e}\right)^{1/2}$$

[T_e & T_i in eV, $A \approx \frac{m_i}{m_p}$ = atomic mass

[v_e & v_i are take as average thermal velocities over a Maxwellian distribution]

Also, r_{Le} & r_{Li} may be expressed in terms of accelerating or (ionizing) potential to substitute for v:

$$\frac{1}{2} m v^2 = eE$$

then for an electron:

$$\underset{(cm)}{r_{Le}} = \frac{v_e}{\omega_{ce}} = \frac{\sqrt{\frac{2e}{m} E}}{\frac{eB}{m}} \cong 3.37 \frac{\sqrt{E_{volt}}}{B_{gauss}}$$

and Similarly:

$$\underset{(cm)}{r_{Li}} \approx 1.44 \times 10^2 \frac{\sqrt{A_R E_{volt}}}{B_{gauss}}$$

and $A_R \equiv$ ion mass in amu