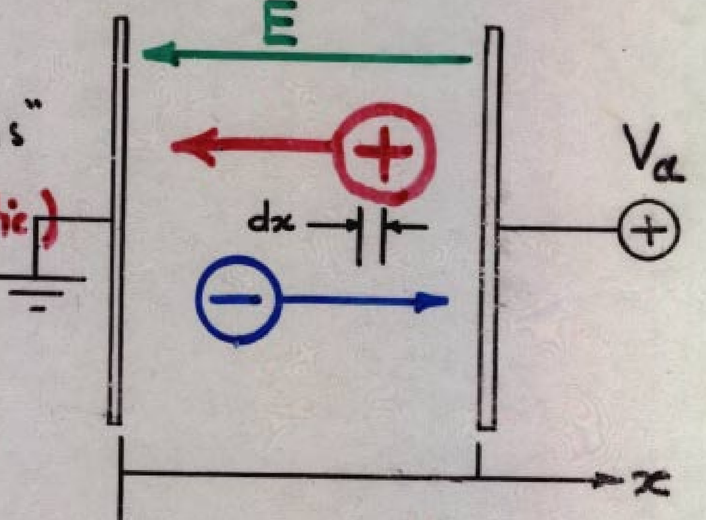
PLASMA PHYSICS

Motion of Charged Particles

Charged particles in Electric Fields :

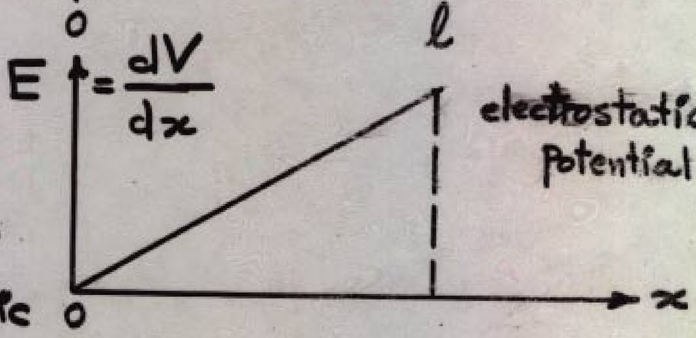
$\bar{F} = q\,\bar{E}$ "Newtons"

$= m \dfrac{d\bar{v}}{dt}$ (electrostatic)



Work done during a small distance dx :

$$dW = \bar{F}.\,d\bar{x}$$
$$= m\,\frac{d\bar{v}}{dt}.\,d\bar{x}$$
$$= m\;\bar{v}.\,d\bar{v} \quad \text{(motion)}$$



if the particle moves from x_1 to x_2, between the electrostatic potentials V_1 and V_2

From the "motion" : $\bar{F}.\,d\bar{x} = m\bar{v}.\,d\bar{v}$

From the "electrostatic" : $\bar{F}.\,d\bar{s} = -q\,dV = -q\underbrace{\frac{dV}{dx}}_{}\!\!{}^{\bar{E}}\,dx$

$$\int_{V_1}^{V_2} \bar{F}.\,d\bar{s} = -q\,(V_2 - V_1) = q\,(V_1 - V_2)$$

$$\int_{x_1}^{x_2} \bar{F}.\,d\bar{x} = \tfrac{1}{2}m\,(v_2^{\,2} - v_1^{\,2})$$; So by Conservation :

$$q\,(V_1 - V_2) = \tfrac{1}{2}m\,(v_2^{\,2} - v_1^{\,2})$$

i.e.

$$\frac{1}{2}mv_1^2+qV_1 = \frac{1}{2}mv_2^2+qV_2 = \underbrace{\text{constant}}_{\substack{\text{Total Energy of Motion} \\ W}}$$

hence:

$$W = \frac{1}{2}mv^2 + qV$$

Consider an electron of charge q is emitted from the cathode, So it's potential energy will be : $qV_a \longrightarrow$ moves towards the anode with K.E. $\frac{1}{2}mv^2$; hits the anode.

So: $$E = eV$$

$$\left[\!\left[1\,eV \cong 1.6 \times 10^{-19}\,J \right]\!\right]$$

and

For an ion: $E = \underbrace{Ze}_{q_i}V$

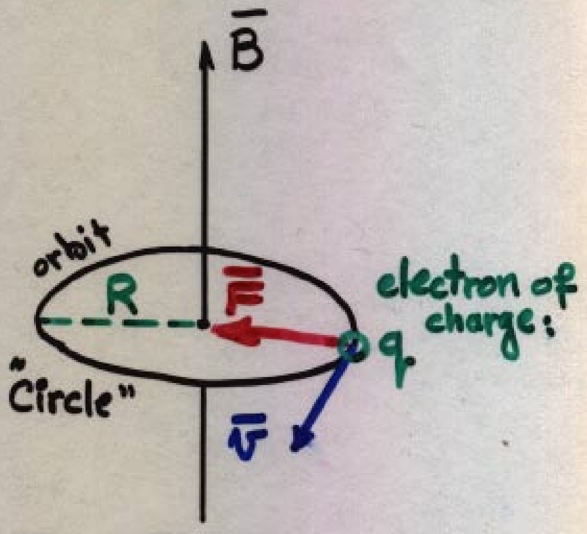
Result: a charged particle will move with a velocity v, and will be accelerated under the influence of an electrostatic field. The motion will be in the direction of the electic field $\overline{E}$.

Charged Particles in Constant Magnetic Field:

$\bar{B}$ ≡ Magnetic Induction

$\bar{H}$ ≡ Magnetic Field Strength

$\bar{B} = \mu \bar{H}$; μ ≡ magnetic permeability



$$\bar{F} = q(\bar{v} \times \bar{B})$$ Lorentz Force

So: will not change the kinetic energy but only the direction ⟵ $\bar{F} \perp \bar{B}$ & $\bar{v}$

Hence: The particle will GYRATE

$\bar{v}$ will remain constant, and centripetal magnetic force will be balanced to the centrifugal force $m\frac{v^2}{R}$, i.e.

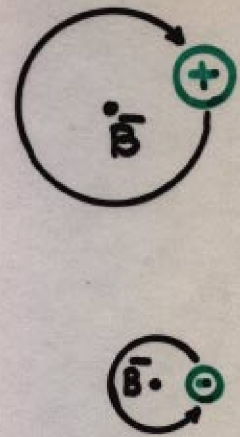
$$q\,\bar{v} \times \bar{B} = m\frac{v^2}{R}$$ "Newtons"

So: the radius of gyration $R = \frac{m}{q}\frac{v^2}{(\bar{v} \times \bar{B})} \equiv \frac{mv}{qB}$ ⟵ [Larmor Radius]

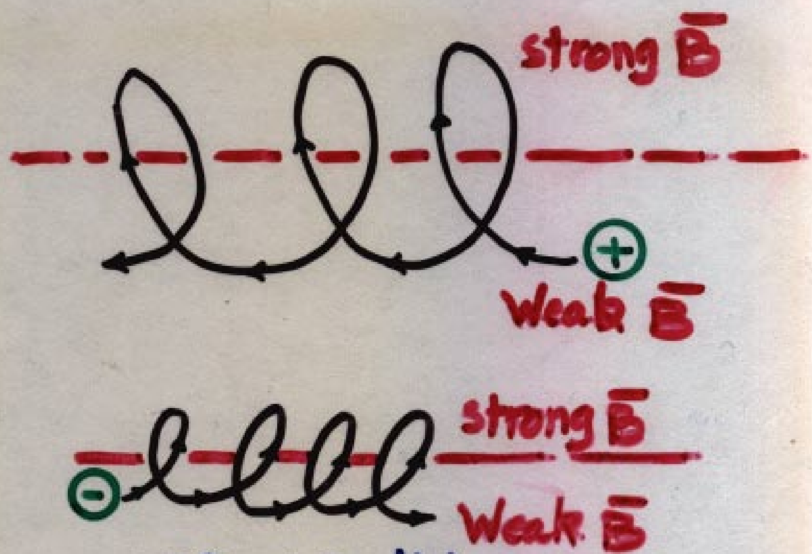
the angular velocity $= \frac{v}{R} = \frac{qB}{m} \equiv \omega$ "gyrofrequency" radians/sec

and the gyrofrequency $\nu = \frac{\omega}{2\pi}$ "Hertz" or "sec^{-1}"

$$= \frac{qB}{2\pi m}$$



+ve & -ve particles in a UNIFORM $\bar{B}$

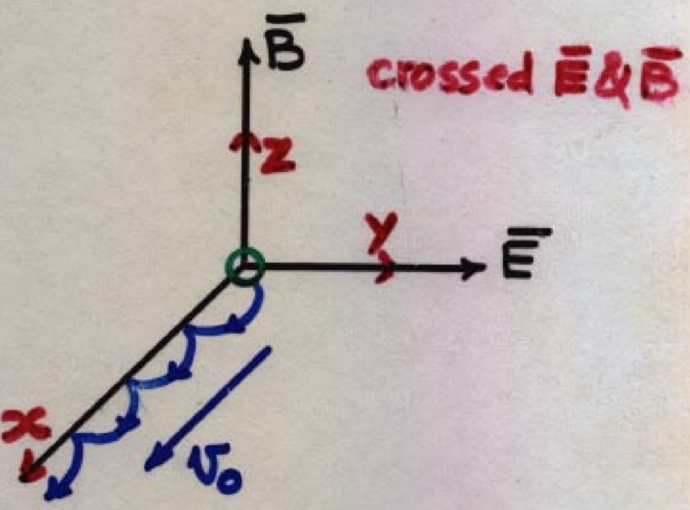


+ve & -ve particles in a GRADIENT $\bar{B}$

"$\nabla \bar{B}$"

Motion in Steady Electric and Magnetic Fields

$$\bar{F} = \underbrace{q\bar{E}}_{\text{electric}} + \underbrace{q(\bar{v} \times \bar{B})}_{\text{magnetic}}$$



$$m\frac{dv_x}{dt} = q(\bar{v} \times \bar{B})_x = qv_yB$$

$$m\frac{dv_y}{dt} = q[\bar{E} + \bar{v} \times \bar{B}]_y = qE - qv_xB$$

$$m\frac{dv_z}{dt} = 0 \quad (\text{i.e. } v_z = \text{const.}, \text{ i.e. particle is confined in } x\text{-}y)$$

$$\frac{dv_x}{dt} = qv_y\frac{B}{m} = \omega v_y = \omega\frac{dy}{dt} \quad \therefore \ v_x = \omega y$$

$$m\frac{dv_y}{dt} = qE - qB\omega y$$

$$\frac{d^2y}{dt^2} = \frac{q}{m}E - qB\omega y \cdot \frac{1}{m} = \frac{qE}{m} - \omega^2 y$$

$$\omega$$

SO: $\dfrac{d^2y}{dt^2} + \omega^2 y = \dfrac{q}{m}E$ .... oscillatory

Solution: $y = \dfrac{qE}{m\omega^2} + K_1 \cos\omega t + K_2 \sin\omega t$

$\|$ K_1, K_2 are constants

$$\frac{dy}{dt} = v_y = -\omega K_1 \sin\omega t + \omega K_2 \cos\omega t$$

iF: +ve particle starts at the origin with $v_{0x} = v_{0y} = 0$ at $t=0$

Hence: $K_2 = 0$; $K_1 = -\dfrac{Eq}{m\omega^2}$

$$y = \frac{qE}{m\omega^2}(1 - \cos\omega t)$$

Cycloide

and We have $v_x = \omega y$

$$= \frac{qE}{m\omega}(1 - \cos\omega t)$$

$$x = \frac{qE}{m\omega^2}(\omega t - \sin\omega t)$$

and: $v_x = \dfrac{E}{B}(1 - \cos\omega t)$ $\|$ average value in the x-direction $= \dfrac{E}{B} = v_d$

$v_y = \dfrac{E}{B}\sin\omega t$ $\|$ average value in the y-direction $= 0$

$$\frac{E}{B} = v_d = \text{Drift Velocity}$$

To generalize, let us use the vector form:

$$\underbrace{m\frac{d\bar{v}}{dt}} = q(\bar{E} + \bar{v}\times\bar{B})$$

could be set to zero as it has nothing to do with the drift velocity, and only gives the circular motion.

SO: $\bar{E} + \bar{v}\times\bar{B} = 0 \quad \times\bar{B}$

$$\bar{E}\times\bar{B} = \bar{B}\times(\bar{v}\times\bar{B})$$
$$= \bar{v}B^2 - \bar{B}(\bar{v}\cdot\bar{B})$$

$\bar{v}\perp\bar{B}$; so we take the transverse component.

$$\bar{v} = \frac{\bar{E}\times\bar{B}}{B^2} \; ; \quad v_d = \frac{E}{B} \qquad ; \; \bar{F} = q\bar{E}$$

$$= \frac{1}{q}\frac{\bar{F}\times\bar{B}}{B^2}$$

if $\bar{F}$ is due to electric field: $\rightarrow q\bar{E}$

if $\bar{F}$ is due to gravitational field: $\rightarrow m\bar{g}$

SO: effect of gravitational field will be:

$$\bar{v} = \frac{m}{q}\frac{\bar{g}\times\bar{B}}{B^2}$$

if There is a gradient in the magnetic field ∇B:

$\nabla B \perp B$: $\bar{v}_{\nabla B} = \pm\frac{1}{2}v_\perp R\frac{\bar{B}\times\bar{\nabla}B}{B^2}$ $\Longrightarrow$

if CURVED B : $\bar{v}_R = \frac{mv_{\parallel}^2}{q}\frac{\bar{R}_c\times\bar{B}}{R_c^2 B^2}$ || R_c = radius of curvature

Total drift in curved-gradient $\bar{B}$:

$$\bar{v} = \bar{v}_{\nabla B} + \bar{v}_R$$

$$= \frac{1}{2} v_\perp R \underbrace{\frac{\bar{B} \times \nabla \bar{B}}{B^2}}_{\frac{1}{B^2}\bar{B} \times |B| \frac{\bar{R}_c}{R_c^2}} + \frac{m}{q} v_\parallel^2 \frac{\bar{R}_c \times \bar{B}}{R_c^2 B^2}$$

$$\frac{|B|}{R_c^2 B^2} \bar{R}_c \times \bar{B}$$

$$\rightarrow \frac{R|B|}{R_c^2 B^2} (\bar{R}_c \times \bar{B}) \quad ; \; R = \frac{m v_\perp}{qB}$$

$$\downarrow$$

$$RB = \frac{m v_\perp}{q}$$

$$= \frac{1}{2} v_\perp \frac{m v_\perp}{q} \cdot \frac{\bar{R}_c \times \bar{B}}{R_c^2 B^2} + \frac{m}{q} v_\parallel^2 \frac{\bar{R}_c \times \bar{B}}{R_c^2 B^2}$$

$$= \frac{m}{q} \cdot \left(v_\parallel^2 + \frac{1}{2} v_\perp^2\right) \frac{\bar{R}_c \times \bar{B}}{R_c^2 B^2} \quad \text{[General]}$$

if $\bar{E}$ is non-uniform: $\bar{v}_E = \left(1 + \frac{1}{4} R^2 \nabla^2\right) \frac{\bar{E} \times \bar{B}}{B^2}$

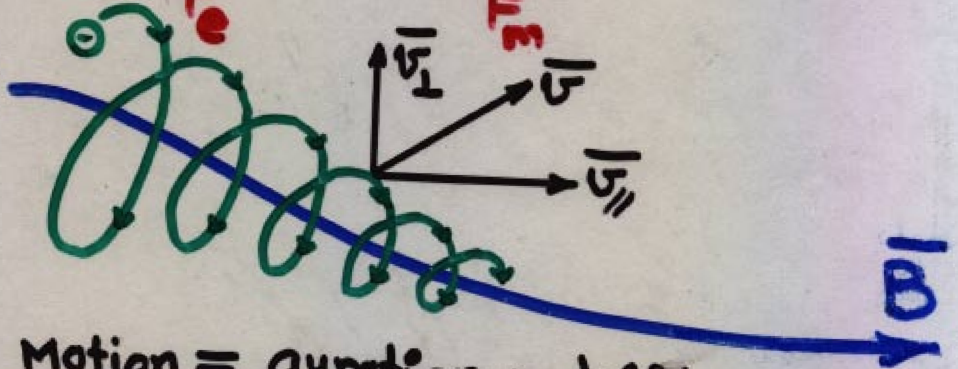
if E is time-varying: $\bar{v}_p = \pm \frac{1}{\omega B} \frac{d\bar{E}}{dt}$

Exact proof of each case "important".

More Deep Understanding of The Drift Velocities

General Force : Crossed Electric & Magnetic Fields:

$$\bar{F} = \underbrace{q\bar{E}}_{\substack{\text{electric}\\ \text{force}\\ \bar{F}_e}} + \underbrace{q(\bar{v} \times \bar{B})}_{\substack{\text{magnetic}\\ \text{force}\\ \bar{F}_m}}$$



Motion ≡ gyration + drift

$$\bar{v}_\perp = \bar{v}_g + \bar{v}_d$$

So:

the magnetic force $\bar{F}_m = q(\bar{v}_\perp \times \bar{B})$

$$= q(\bar{v}_g + \bar{v}_d) \times \bar{B}$$

$$= \underbrace{q(\bar{v}_g \times \bar{B})}_{\bar{F}_g} + \underbrace{q(\bar{v}_d \times \bar{B})}_{\bar{F}_d}$$

hence: GENERALLY:

Drift Force $\bar{F}_d = q(\bar{v}_d \times \bar{B})$

so: for any external force $\bar{F}_{ext}$: which will cause drift

$$\bar{F}_{ext} \times \bar{B} = q(\bar{v}_d \times \bar{B}) \times \bar{B}$$
$$= [B^2 v_d - \underbrace{(\bar{v}_d \cdot \bar{B})}_{=0}\bar{B}]\, q$$

from which:

$$v_d = \frac{1}{q}\frac{\bar{F}_{ext} \times \bar{B}}{B^2} \equiv$$ Generalized drift velocity due to any external force

NOW: what are the different forms of $\bar{F}_{ext}$??

<u>Case I:</u> <u>$\bar{F}_{ext}$ is ELECTRIC</u>

$$\bar{F}_{ext} = \bar{F}_e = q\bar{E}$$ from which:

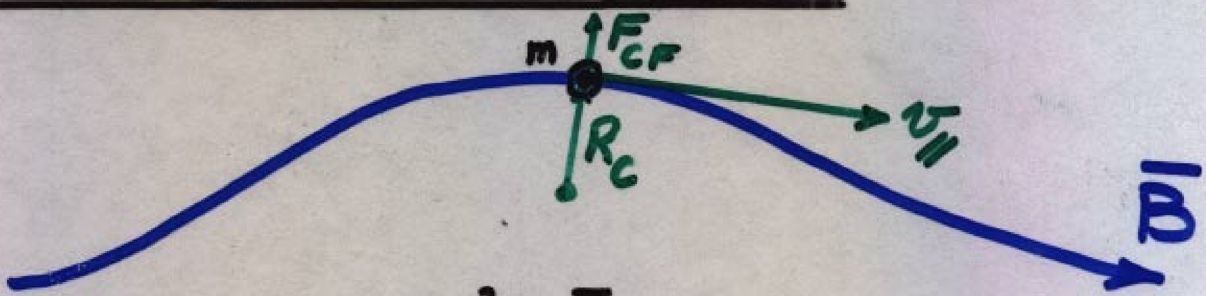
$$v_{d(E)} = \frac{q}{q}\frac{\bar{E} \times \bar{B}}{B^2} = \frac{E_\perp}{B}$$

<u>Case II:</u> <u>$\bar{F}_{ext}$ is GRAVITATIONAL</u>

$$\bar{F}_{ext} = \bar{F}_{grav.} = m\bar{g}$$ from which:

$$v_{d(grav.)} = \frac{m}{q}\frac{\bar{g} \times \bar{B}}{B^2}$$
$$= \frac{m}{q}\frac{g_\perp}{B}$$

case III : $\bar{F}_{ext}$ CENTRIFUGAL



$$\bar{F}_{ext} = \bar{F}_{CF} = -\frac{m v_{\parallel}^2}{R_c} \cdot \bar{k}$$

$$\bar{k} \downarrow \frac{\bar{R}_c}{R_c}$$

$\bar{k} \equiv$ unit vector along radius of curvature

$$= -\frac{m v_{\parallel}^2}{R_c^2} \bar{R}_c$$

$$v_{d(CF)} = \frac{1}{qB^2}\left(-\frac{m v_{\parallel}^2}{R_c^2} \bar{R}_c \times \bar{B}\right) = \frac{m v_{\parallel}^2}{q} \frac{\bar{B} \times \bar{R}_c}{B^2 R_c^2}$$

Called : Centrifugal OR Curvature drift

and more generally : $\dfrac{\bar{k}}{R_c} = \dfrac{(\bar{B} \cdot \bar{\nabla})\bar{B}}{B^2} = \dfrac{\bar{R}_c}{R_c^2}$

and so:

$$v_{d(CF)} = \frac{m v_{\parallel}^2}{q B^4}\left[\bar{B} \times (\bar{B} \cdot \bar{\nabla})\bar{B}\right]$$

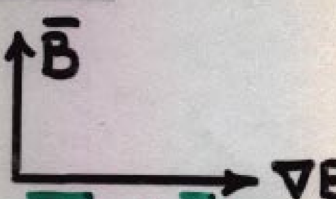
$$= \frac{q}{m}\left(\frac{v_{\parallel}}{v_{\perp}}\right)^2 R^2 \left[\bar{B} \times (\bar{B} \cdot \bar{\nabla})\bar{B}\right] \frac{1}{B^2}$$

$$= \frac{q}{m}\left(\frac{v_{\parallel}}{v_{\perp}}\right)^2 \left(\frac{R}{R_c}\right)^2 (\bar{B} \times \bar{R}_c) \quad ;$$

$R = \dfrac{m v_{\perp}}{qB}$ = gyration radius.

Case IV: $\bar{F}_{ext}$ due to GRADIENT of B

$$\bar{F}_{ext} = \bar{F}_{\nabla B \perp} = \underbrace{-\tfrac{1}{2} m v_\perp^2}_{\text{Kinetic energy}} \frac{\nabla B}{B}$$



$\overline{\nabla B} \perp \bar{B}$

so:

$$v_{d(\nabla\perp)} = \frac{1}{q}\left(-\frac{1}{2} m v_\perp^2 \frac{\overline{\nabla B}}{B} \times \bar{B}\right)\frac{1}{B^2}$$

$$= \frac{m v_\perp^2}{2qB} \frac{\bar{B} \times \overline{\nabla B}}{B^2} = \frac{1}{2} R v_\perp \frac{\bar{B} \times \overline{\nabla B}}{B^2}$$

R

more generalized: $\nabla B = \frac{1}{2} \frac{\nabla B^2}{B}$

so:

$$v_{d(\nabla\perp)} = \frac{m v_\perp^2}{2qB} \cdot \frac{1}{B^2} \cdot \left(\bar{B} \times \frac{\nabla B^2}{B}\right) \cdot \frac{1}{2}$$

$$= \frac{m v_\perp^2}{4qB^4} (\bar{B} \times \nabla B^2)$$

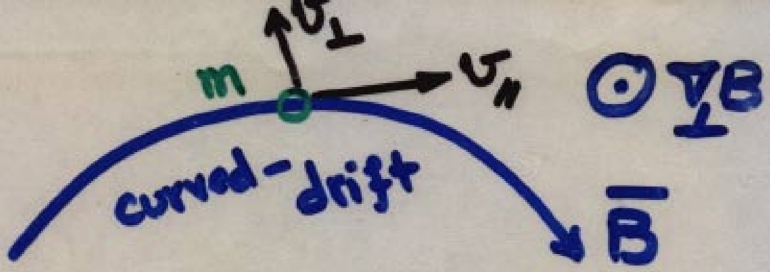
$$= \frac{R v_\perp}{4B^3} (\bar{B} \times \nabla B^2)$$

what does it mean to have both curved and gradient drift?! :

$$v_{CF.\nabla\perp} = v_{d(CF)} + v_{d(\nabla\perp)} = \frac{m}{q}\left(v_\parallel^2 + \frac{1}{2} v_\perp^2\right) \frac{\bar{B} \times \bar{R}_c}{B^2 R_c^2}$$

$$\text{or } = \frac{m}{qB^4}\left(v_\parallel^2 + \frac{1}{4} v_\perp^2\right)\left[\bar{B} \times \overline{(B \cdot \nabla) B} + \bar{B} \times \overline{\nabla B^2}\right]$$

AND BOTH DRIFTS ARE OUTWARDS $\Rightarrow$

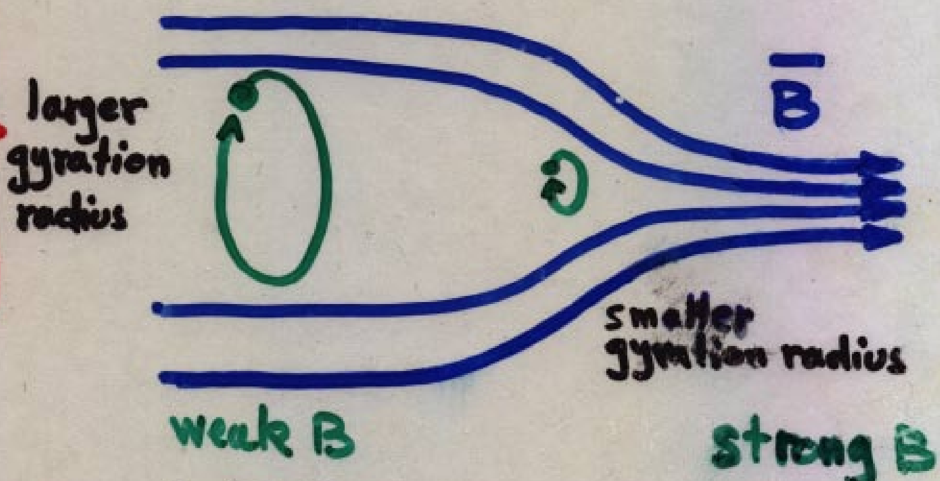


So, if you bend the magnetic field lines, the particles will drift outwards " in addition to the drift caused by the electric field which will be induced due to the charge separation ".

Case V: $\bar{F}_{ext}$ due to GRADIENT of B

BUT $\nabla B // B$

That's the principle of the magnetic mirror



$$\bar{F}_{ext} = -\frac{1}{2} m v_\perp^2 \frac{\nabla_\parallel B}{B} = -\frac{1}{2} q(\underbrace{R\, \nabla_\parallel B}_{\text{variable radius of gyration}})\, v_\perp$$

$$= -M\, \nabla_\parallel B$$

M = magnetic moment (or sometimes denoted by μ)

What is the meaning of the magnetic moment?

The magnetic moment is the current flowing arround the boundary of the gyration orbit X area and is approximately constant $\Longrightarrow$

so: if the particle is gyrating within a circle of radius $= R =$ gyro-radius:

$$\text{area} = \pi R^2$$

So: $$M = I(\pi R^2) = I \frac{\pi m^2 v_\perp^2}{q^2 B^2}$$

and the current produced due to a gyrating particle is: $I = q\nu$ || ν = frequency

$$= q\frac{\omega_c}{2\pi}$$

$$= q\frac{qB}{2\pi m} = \frac{q^2 B}{2\pi m}$$

hence M:

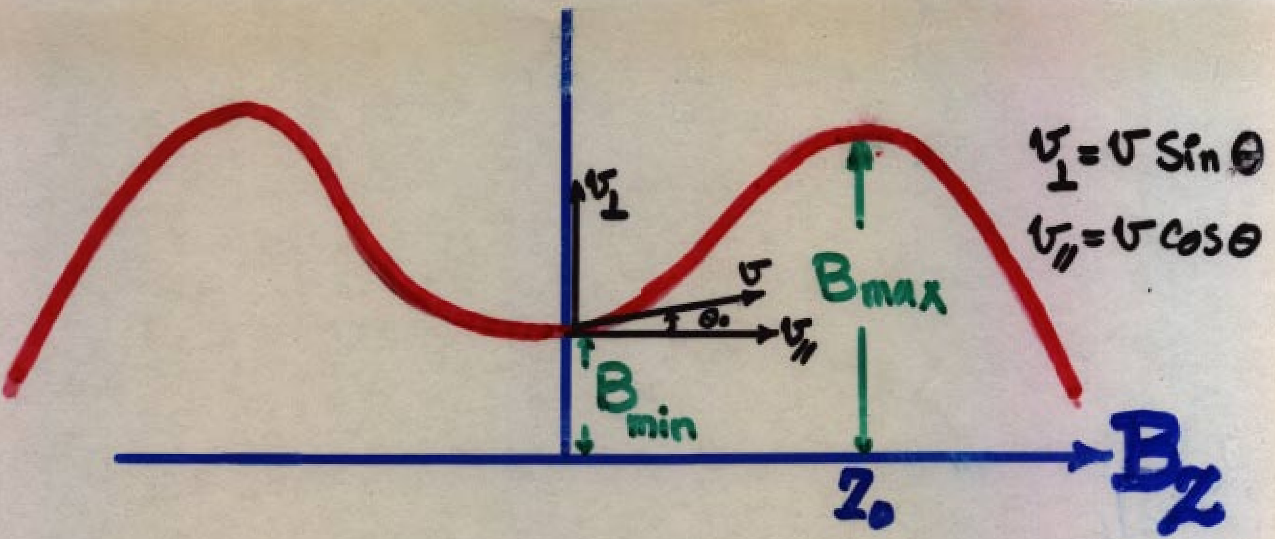
$$= \frac{q^2 B}{2\pi m} \cdot \frac{\pi m^2 v_\perp^2}{q^2 B^2} = \frac{1}{2} m \frac{v_\perp^2}{B}$$

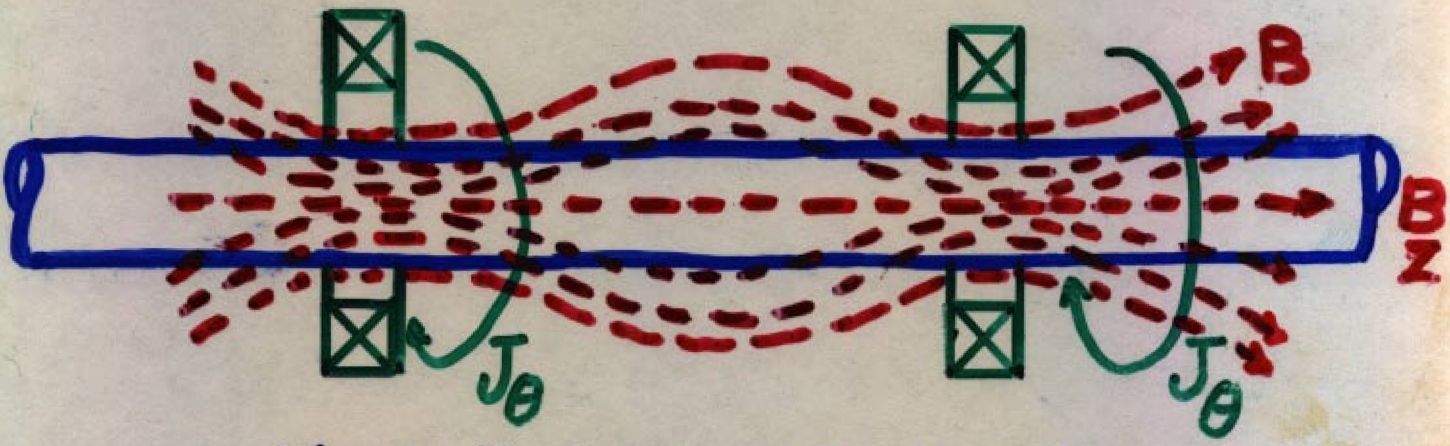
as the magnetic moment $M \approx$ Constant this means that: $\overline{F}_{ext} = \overline{F}_{\nabla_{\parallel}} = -M \nabla_{\parallel} B$

$$\propto \nabla_{\parallel} B$$

So: if the magnetic field is strong, $v_{\parallel} \rightarrow 0$ and the particle will reflect

i.e. The particle start it's motion from a weak B field with higher $v_\perp$ and lower $v_{\parallel}$ and then reaches the strong B region where it's $v_{\parallel}$ approaches zero and then reflects back to the weak region.





Magnetic Mirror Configuration

Towards more understanding:

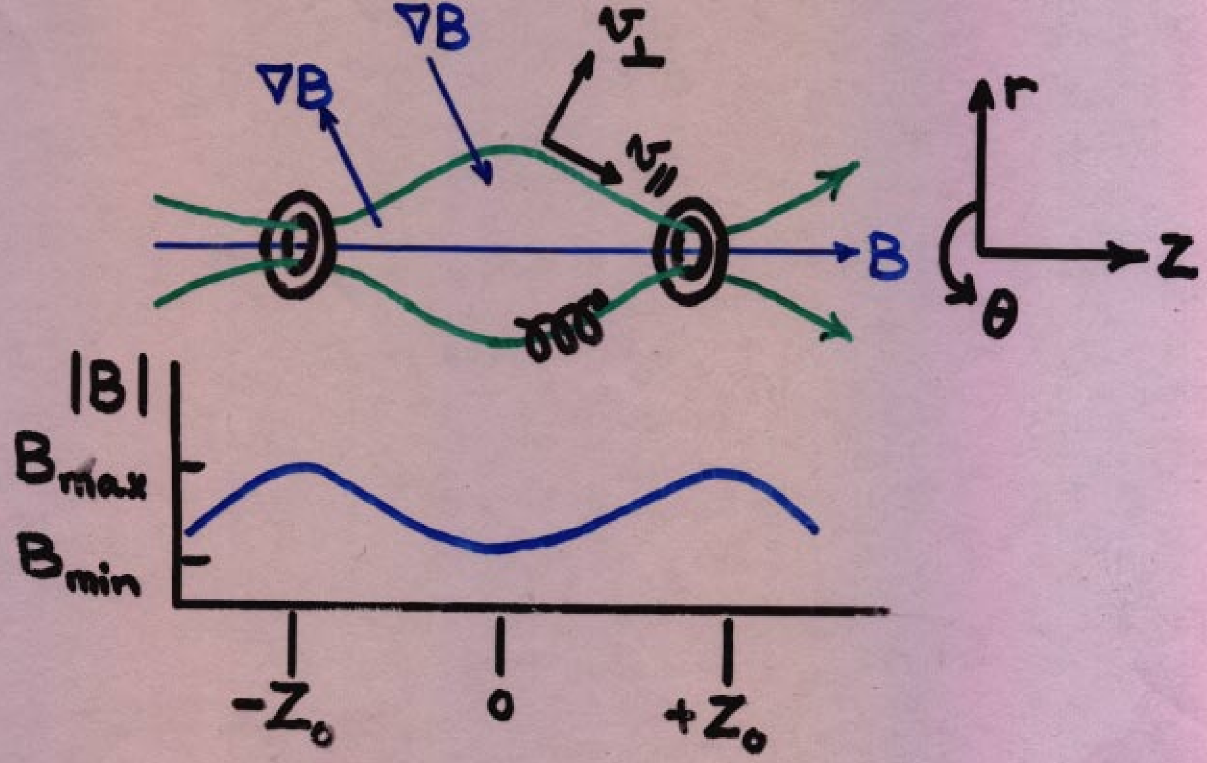
As M "the magnetic moment" is constant,

So: $$M\Big|_{z=0} = M\Big|_{z=z_0}$$

i.e: $$M\Big|_{B_{min}} = M\Big|_{B_{max}}$$

$$\frac{1}{2} m \frac{v^2 \sin^2\theta_0}{B_{min}} = \frac{1}{2} m \frac{\left.v_\perp^2\right\}}{B_{max}} \longrightarrow$$

at Z_0, $v_\parallel = 0$ and $v_\perp = v$ "for Confinement"



magnetic mirror Configuration

- close to Coils ∇B outward
- close to midplane ∇B inward
- B-field maximum at Coils
  minimum at midplane
- $v_\parallel \to 0$ at $\pm Z_0$

So: $\sin\theta_o = \left(\frac{B_{min}}{B_{max}}\right)^{1/2}$ and is called: Magnetic Mirror Ratio

if: $\frac{B_{min}}{B_{max}} \ll \longrightarrow \theta_o \ll$ and better confinement

if: $\frac{B_{min}}{B_{max}} = 1 \longrightarrow \theta_o = \frac{\pi}{2}$ and particle loss will be increased as any scattering angle of $\frac{\pi}{2}$ will lead to escaping of the particle

Applications:

- Open-ended Systems "Fusion"
  e.g. θ-pinches, Z-pinches

NOTE 1: The effect works on both ions and electrons

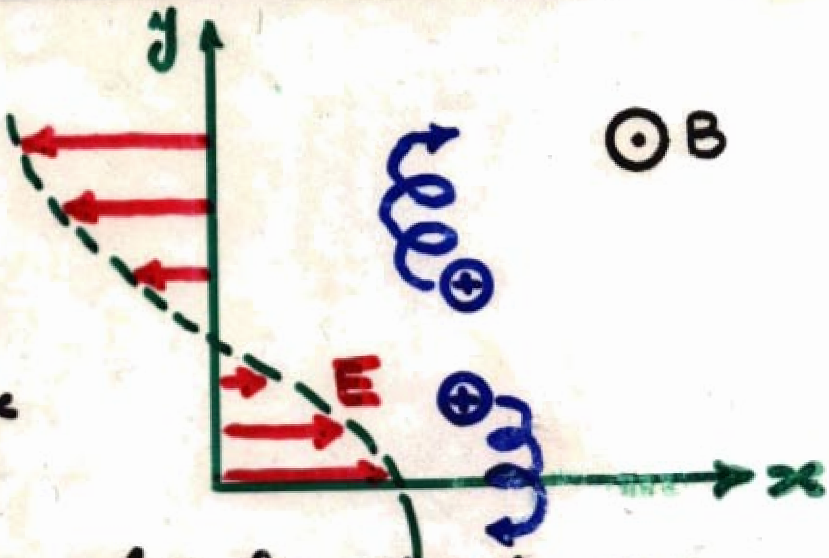
NOTE 2: The trapping is not perfect
e.g. a particle with $\frac{v_\perp}{v_\parallel} \ll$ at $Z=0$
i.e. at B_{min} WILL ESCAPE
if B_{max} is not large enough.

Case VI $\bar{F}_{ext}$ ELECTRIC non-uniform

case VI

Assume:

$$\bar{E} \equiv E_x(y)$$
$$= \left[E_0 \cos ky\right]_x$$



WHY Sinusoidal !?

Because such field can arrise from the charge distribution in the plasma during a wave motion.

Now: eqn of motion:

$$m\frac{d\bar{v}}{dt} = q\left[\bar{E} + (\bar{v} \times \bar{B})\right]$$

from which;

$$m\frac{dv_x}{dt} = q\,E_x(y) + q\,v_y B \quad \ldots\ldots(1)$$

$$m\frac{dv_y}{dt} = -q\,v_x B \quad \ldots\ldots\ldots\ldots(2)$$

differentiating:

$$m\frac{d^2v_x}{dt^2} = q\,\frac{dE_x(y)}{dt} + qB\frac{dv_y}{dt} \quad \ldots(3)$$

$$m\frac{d^2v_y}{dt^2} = -qB\frac{dv_x}{dt} \quad \ldots\ldots\ldots(4)$$

(2) into (3) yields:

$$\frac{d^2 v_x}{dt^2} = \underbrace{\frac{q}{m}}_{\omega_c/B} \frac{dE_x(y)}{dt} - \underbrace{\frac{q^2 B^2}{m^2}}_{\omega_c^2} v_x$$

$$= \frac{\omega_c}{B} \frac{dE_x(y)}{dt} - \omega_c^2 v_x \quad \text{......} (5)$$

and Consequently: (1) into (4) yields:

$$\frac{d^2 v_y}{dt^2} = -\omega_c^2 \left(\frac{1}{B} E_x(y) + v_y \right) \quad \text{......} (6)$$

Consider that the electric field is WEAK; then the value of y could be expressed as:

$$y = \frac{qE}{m\omega_c^2} (1 - \cos \omega_c t)$$

$$= \frac{1}{\omega_c^2} \cdot (1 - \cos \omega_c t) \cdot \frac{qE}{m} \cdot \frac{B}{B}$$

$\rightarrow \omega_c$

$\rightarrow v_{Drift}$

$$= \underbrace{\frac{v}{\omega_c}}_{R} (1 - \cos \omega_c t)$$

R "radius of gyration" (undisturbed orbit")

$$= R (1 - \cos \omega_c t) \quad \text{..........} (7)$$

back to eqn (6)

$$\frac{d^2 v_y}{dt^2} = -\omega_c^2 \left[v_y + \frac{1}{B} \underbrace{E_x(y)} \right]$$

$$\downarrow$$

$E_0 \cos ky$ from (7)

$y \rightarrow R(1-\cos\omega_c t)$

$$\downarrow$$

$$\underbrace{E_0 \left\{ \cos kR(1-\cos\omega_c t) \right\}}$$

EXPANDING

$$\cos Rk \cos(Rk\cos\omega_c t) + \sin Rk \sin(Rk\cos\omega_c t)$$

Now: for small $\underline{\underline{R}}$

$$\cos(Rk\cos\omega_c t) \approx 1 - \frac{1}{2}R^2k^2\cos^2\omega_c t$$

and $\sin(Rk\cos\omega_c t) \approx Rk\cos\omega_c t$

from which:

$$\cos Rk\left[1 - \frac{1}{2}R^2k^2\cos^2\omega_c t\right] + Rk\cos\omega_c t \sin Rk$$

if to average over one complete cycle:-
yields:

$$\cos Rk\left[1 - \frac{1}{2}R^2k^2\right] - 0$$

as we averaged over one complete cycle, let us re-arrange our equation:

$$\underbrace{\left\langle \frac{d^2 v_y}{dt^2} \right\rangle}_{0} = -\omega_c^2 \langle v_y \rangle + \omega_c^2 \frac{E_0 \cos Rk}{B} \cdot \left(1 - \frac{1}{4} R^2 k^2\right)$$

$\to E_x(y_0)$

"oscillating term"

$$0 = -\omega_c^2 \langle v_y \rangle + \omega_c^2 \frac{E_x(y_0)}{B} \cdot \left(1 - \frac{1}{4} R^2 k^2\right)$$

$\to \dfrac{\bar{E} \times \bar{B}}{B^2}$

$$0 = -\omega_c^2 \langle v_y \rangle + \omega_c^2 \frac{\bar{E} \times \bar{B}}{B^2} \cdot \left(1 - \frac{1}{4} R^2 k^2\right)$$

from which:

$$\underbrace{\langle v_y \rangle}_{\text{The new drift velocity}} = \frac{\bar{E} \times \bar{B}}{B^2} \left(1 - \frac{1}{4} R^2 k^2\right)$$

$k^2 \to (jk)^2$

$$v_{drift} = \frac{\bar{E} \times \bar{B}}{B^2} \left(1 + \frac{1}{4} R^2 \underbrace{(jk)^2}_{\nabla^2}\right)$$

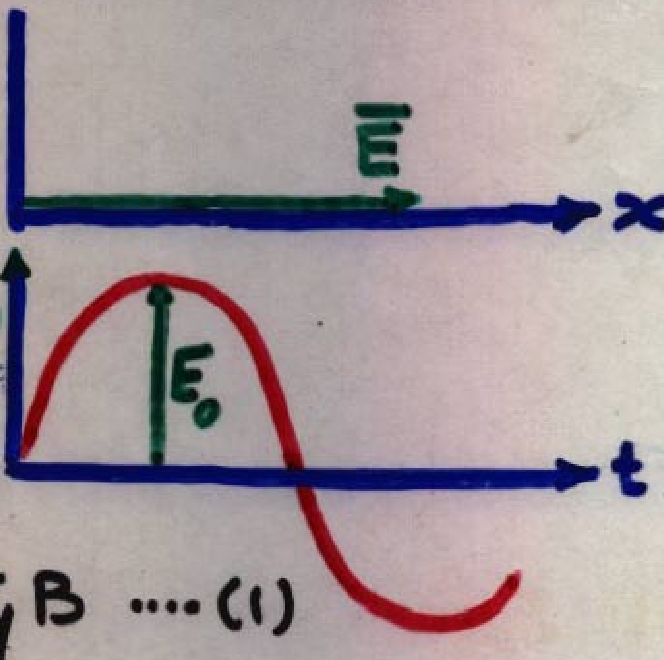
∇^2 for any arbitrary variation in E

and finally:

$$v_{d(\text{non-uniform } E)} = \left(1 + \underbrace{\frac{1}{4} R^2 \nabla^2}_{\text{finite Larmor radius effect}}\right) \frac{\bar{E} \times \bar{B}}{B^2}$$

Case VII : $\overline{F}_{ext}$ ELECTRIC time-varying

$$E_x = E_o e^{j\omega t}$$



NOW; for the drift velocity: let us use the equations of motion:

$$m\frac{d v_x}{dt} = qE_x + q v_y B \quad \text{....(1)}$$

and $$m\frac{d v_y}{dt} = -q v_x B \quad \text{......(2)}$$

from (1): by differentiation:

$$\frac{d^2 v_x}{dt^2} = \frac{q}{m}\frac{dE_x}{dt} + \frac{qB}{m}\frac{d v_y}{dt}$$

$$\frac{qB}{mB} = \frac{\omega_c}{B}$$

$$j\omega E_o e^{j\omega t} = j\omega E_x$$

$$= \frac{\omega_c}{B} j\omega E + \omega_c \frac{d v_y}{dt}$$

replace by: $-\frac{qB}{m} v_x = -\omega_c v_x$ from ..(2)

$$= \omega_c j\frac{\omega E}{B} - \omega_c^2 v_x$$

$$= -\omega_c^2\left(v_x - j\frac{\omega}{\omega_c}\frac{E_x}{B}\right)$$

$$= \underbrace{-\omega_c^2 v_x}_{\text{This part represents ordinary oscillatory motion with a frequency } \omega_c \text{ which is the ordinary case}} + \underbrace{\omega_c^2\left[j\frac{\omega}{\omega_c}\frac{E_x}{B}\right]}_{\text{This part represents an additional drift velocity caused by the time-varying electric field, and could be replaced by: } \omega_c^2 v_p}$$

$$= -\omega_c^2(v_x - v_p) \quad \cdots\cdots\cdots (3)$$

Let us find a solution by assuming that the velocity v_x is composed of two velocities:

A drift and a gyration in the form:

$$v_x = \underbrace{v_\perp e^{j\omega_c t}}_{\text{gyration}} + \underbrace{v_p}_{\text{drift}}$$

by differentiation:

$$\frac{d^2 v_x}{dt^2} = -\omega_c^2 v_\perp e^{j\omega_c t} + \underbrace{\frac{d^2 v_p}{dt^2}}_{\underbrace{\frac{d^2}{dt^2}\left(j\frac{\omega}{\omega_c}\frac{E}{B}\right)}_{\underbrace{j\frac{\omega}{\omega_c B}\frac{d^2E}{dt^2}}_{\underbrace{j\frac{\omega}{\omega_c B}\left(-\omega^2 E\right)}_{\underbrace{-\omega^2\left(j\frac{\omega}{\omega_c}\frac{E}{B}\right)}_{-\omega^2 v_p}}}}}$$

$$= -\omega_c^2 \underbrace{v_\perp e^{j\omega_c t}}_{v_x - v_p} - \omega^2 v_p$$

By our definition

$$= -\omega_c^2 (v_x - v_p) - \omega^2 v_p$$

$$= -\omega_c^2\left[v_x - \left(1 - \frac{\omega^2}{\omega_c^2}\right)v_p\right]$$

if our assumption is correct; i.e. our velocity v_x is a composition of two velocities GYRATION + DRIFT, this means that:

$$\underbrace{\left.\frac{d^2 v_x}{dt^2}\right|_{\text{which we obtained}}}_{-\omega_c^2\left[v_x - \left(1 - \frac{\omega^2}{\omega_c^2}\right)v_p\right]} = \underbrace{\left.\frac{d^2 v_x}{dt^2}\right|_{\text{which we have before; eq}^{\underline{n}}\text{(3)}}}_{-\omega_c^2(v_x - v_p)}$$

i.e. $\quad v_p = v_p\left(1 - \dfrac{\omega^2}{\omega_c^2}\right)$

which means that the assumption is correct <u>IF</u> $\dfrac{\omega}{\omega_c} \ll 1$, i.e. if the electric field is slowly varing with time AND it's frequency is much less than the cyclotron frequency, THEN:

$$v_x = v_\perp e^{j\omega_c t} + v_p$$

HENCE:

The assumed new drift velocity due to the time varying electric field is given by:

$$v_p = j\frac{\omega}{\omega_c}\frac{E_x}{B} \rightarrow \frac{dE_x}{dt}$$

$$= \frac{1}{\omega_c B}\frac{dE}{dt} \quad \ldots\ldots\ldots (4)$$

and what about the velocity in the y-direction?

from eq$^{\underline{n}}$ (2):

$$\frac{dv_y}{dt} = -\frac{qB}{m} v_x = -\omega_c v_x$$

$$= -\omega_c \underbrace{\left(v_\perp e^{j\omega_c t} + v_p\right)}_{\text{from our assomption}}$$

$$= -\omega_c \left(v_\perp e^{j\omega_c t} + \underbrace{\frac{1}{\omega_c B}\frac{dE}{dt}}_{\text{from eq}^{\underline{n}}\text{(4)}}\right)$$

$$= -\omega_c v_\perp e^{j\omega_c t} - \frac{1}{B}\frac{dE}{dt}$$

uppon integration:

$$v_y = -\omega_c v_\perp \frac{e^{j\omega_c t}}{j\omega_c} - \frac{E}{B}$$

our drift Velocity due to the Electric field $= v_{d(E)}$

$$= j v_\perp e^{j\omega_c t} - v_{d(E)} = j(v_x - v_p) - v_{d(E)}$$

NOW WE COME TO The meaning:

Y-Component: "The ordinary drift due to the ELECTRIC FIELD"
→ BUT $v_{d(E)}$ oscillates Slowly with a frequency ω, which is the frequency of the electric field.

X-Component: Drift along the E-direction and called "polarization"

So: Polarization drift:

$$v_p = \pm \frac{1}{\omega_c B} \frac{d\bar{E}}{dt}$$

- $\bar{F}$ is due to time-varying $\bar{E}$-field "polarization"

Simple approach:

let $\bar{B}$-field be uniform along z & $= B_0$
let $\bar{E}$-field be along x and $= \hat{x}\,\bar{E}(t)$
and varies slowly in time

$$\therefore \bar{v}_E = \frac{q}{q}\frac{\bar{E}\times\bar{B}}{B^2} = -\frac{E(t)}{B_0}\,\hat{y}$$

$\therefore$ guiding center accelerates along y

$$a(t) = -\frac{1}{B_0}\frac{\partial E}{\partial t}\,\hat{y}$$

Force: (transverse to $\bar{B}$)

$$-F = -ma = F_{\text{Polarization}}$$

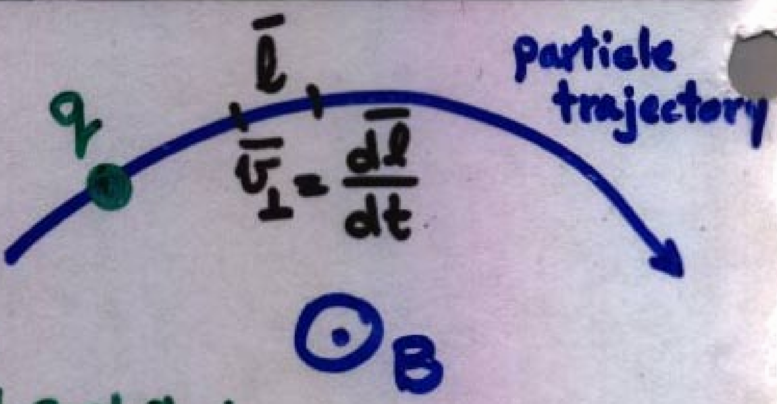
i.e. $$\bar{F}_P = \frac{m}{B_0}\frac{\partial \bar{E}}{\partial t}\,\hat{y}$$

then $$\bar{v}_E = \frac{1}{q}\frac{m}{B_0}\cdot\frac{(\partial\bar{E}/\partial t)\times\bar{B}}{B_0^2} = \frac{m}{qB_0^2}\frac{\partial\bar{E}}{\partial t}$$

i.e. $$|v_E| = \frac{1}{\omega_c B_0}\frac{\partial E}{\partial t}$$

Case VIII : $\bar{F}_{ext}$ MAGNETIC, Time Varying

equation of motion:



$$m\frac{d\bar{v}}{dt} = q\bar{E} + \underbrace{q(\bar{v}\times\bar{B})}$$

does not contribute to give energy to charged particles, but the rate of change of B with time → E by:

$$\left[\nabla\times E = -\frac{\partial B}{\partial t}\right]$$

$$= q\bar{E}$$

multiply the eqn by $v_\perp$ in the scaler form:

$$m\bar{v}_\perp\frac{d\bar{v}_\perp}{dt} = q\,\bar{E}\cdot\bar{v}_\perp$$

OR

$$\frac{d}{dt}\left(\frac{1}{2}mv_\perp^2\right) = q\,\bar{E}\cdot\bar{v}_\perp = q\,\bar{E}\cdot\frac{d\bar{\ell}}{dt}$$

change in K.E. over one periode

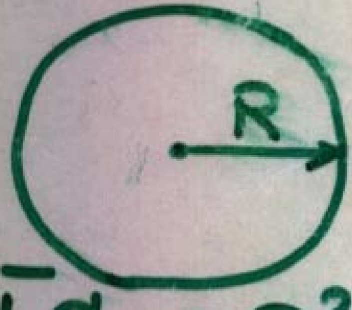
$$\delta\left(\frac{1}{2}mv_\perp^2\right) = \int_0^{2\pi/\omega_e} q\,\bar{E}\cdot\frac{d\bar{\ell}}{dt}\,dt$$

$$= q\int_0^{2\pi/\omega_e}\bar{E}\cdot d\bar{\ell} = q\oint\bar{E}\cdot d\bar{\ell}$$

now: if B is slowly varying, we can replace the time integral by a line integral over the orbit:

i.e. $$\oint \bar{E}\cdot d\bar{\ell} \approx \int_S (\bar{\nabla}\times\bar{E})\cdot d\bar{S}$$

$$= -\int_S \overline{\frac{\partial B}{\partial t}}\cdot d\bar{S}$$



$$S = \int_S d\bar{S} = \pi R^2$$

$$= -\pi R^2 \frac{\partial B}{\partial t}$$ — Sign indicating direction

and then:

$$\delta\left(\tfrac{1}{2} m v_\perp^2\right) = \pi R^2 q \frac{\partial B}{\partial t}$$

$$= \pi q \frac{\partial B}{\partial t}\,\frac{m^2 v_\perp^2}{q^2 B^2}$$

$$= \underbrace{\frac{\left(\tfrac{1}{2} m v_\perp^2\right)}{B}}_{M}\left(\frac{2\pi}{\omega_c}\frac{\partial B}{\partial t}\right)$$

i.e.

$$\underbrace{\delta\left(\tfrac{1}{2}mv_{\perp}^{2}\right)}_{MB} = M\underbrace{\left(\frac{2\pi}{\omega_c}\frac{\partial B}{\partial t}\right)}_{\text{change in B over one complete cycle} = \delta B}$$

$$\delta(MB) = M\,\delta B$$

$$M\delta B + B\delta M = M\delta B$$

i.e. $\delta M = 0 \longrightarrow$ IMPORTANT

$\Downarrow$

magnetic moment is constant in a slowly varying magnetic field.

"INVARIANT"

WHAT does it mean:

AS B varies, the orbit also varies, i.e. Larmor Radius contracts and expands, and the Particle <u>gains</u> or <u>loses</u> transverse energy

This also means that the magnetic flux through the Larmor radius is CONSTANT

Magnetic Flux $\Phi = BS$

$$= B(\pi R^2)$$

$$= \pi B \frac{m^2 v_\perp^2}{q^2 B^2}$$

$$= 2\pi \frac{m}{q^2} \cdot \frac{\frac{1}{2} m v_\perp^2}{B}$$

$$= 2\pi \frac{m}{q^2} M$$

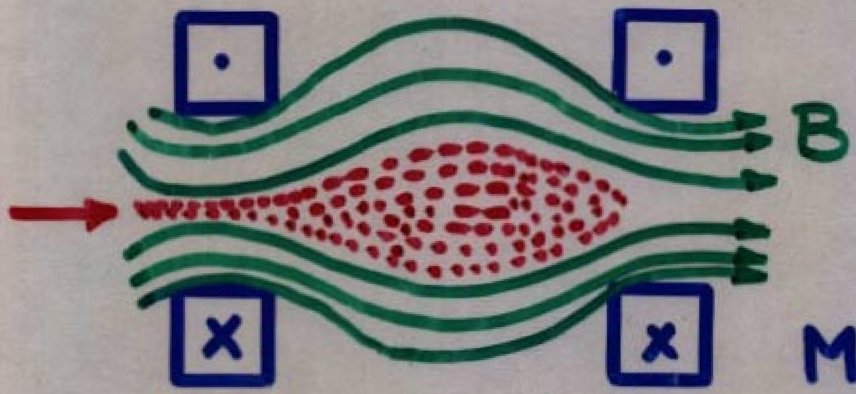
if: M is constant, then $\Phi = \text{Const.}$

So: The drift due to a time-varying magnetic field will cause the <u>expansion</u> and <u>contraction</u> of the radius of gyration "R".

What is the use of the fact that The Magnetic Flux will not change as the Magnetic Moment will remain constant?

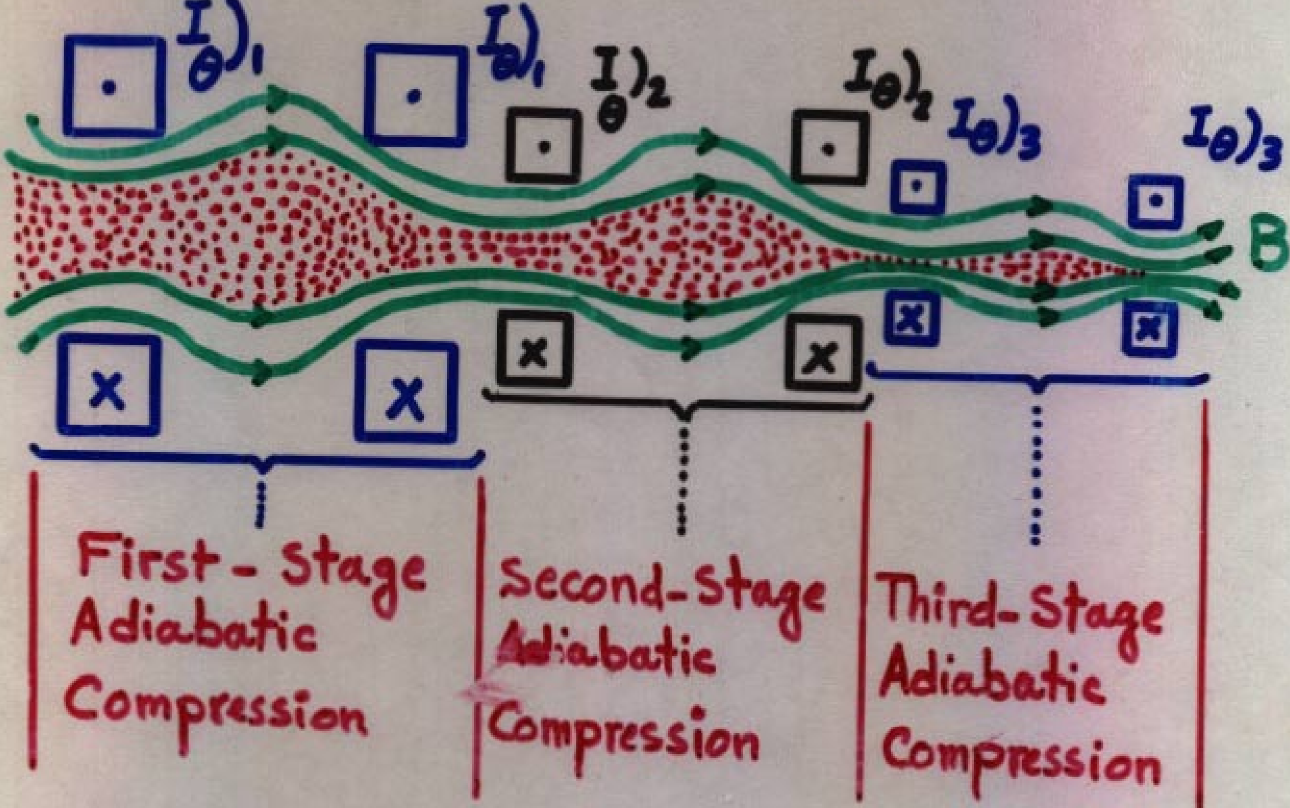
[As M is invariant, this could be used to heat the plasma.]

That's The **Adiabatic Compression**



- Plasma is injected inside The magnetic mirror region **THEN**
- Magnetic Coils are pulsed, i.e. variable B and increasing B-field
- Consequently: $v_\perp$ increases, i.e. $\frac{1}{2} m v_\perp^2$ increases.

Multi-stage Adiabatic Compression:



- All I_θ's are pulsed currents, and consequently B_z's are pulsed, i.e. time varying.
- 1st adiabatic Compression will cause an increase in the kinetic energy of the charged particles as $v_\perp$ will be increased.
- The 1st half of the magnet will be pulsed again, thus the plasma will be transported to the region of the 2nd magnetic region, and the magnetic mirror ratio will be increased.
- The 2nd pulsed current $I_\theta)_2$ will cause a further adiabatic Compression and further increase in $v_\perp$ and

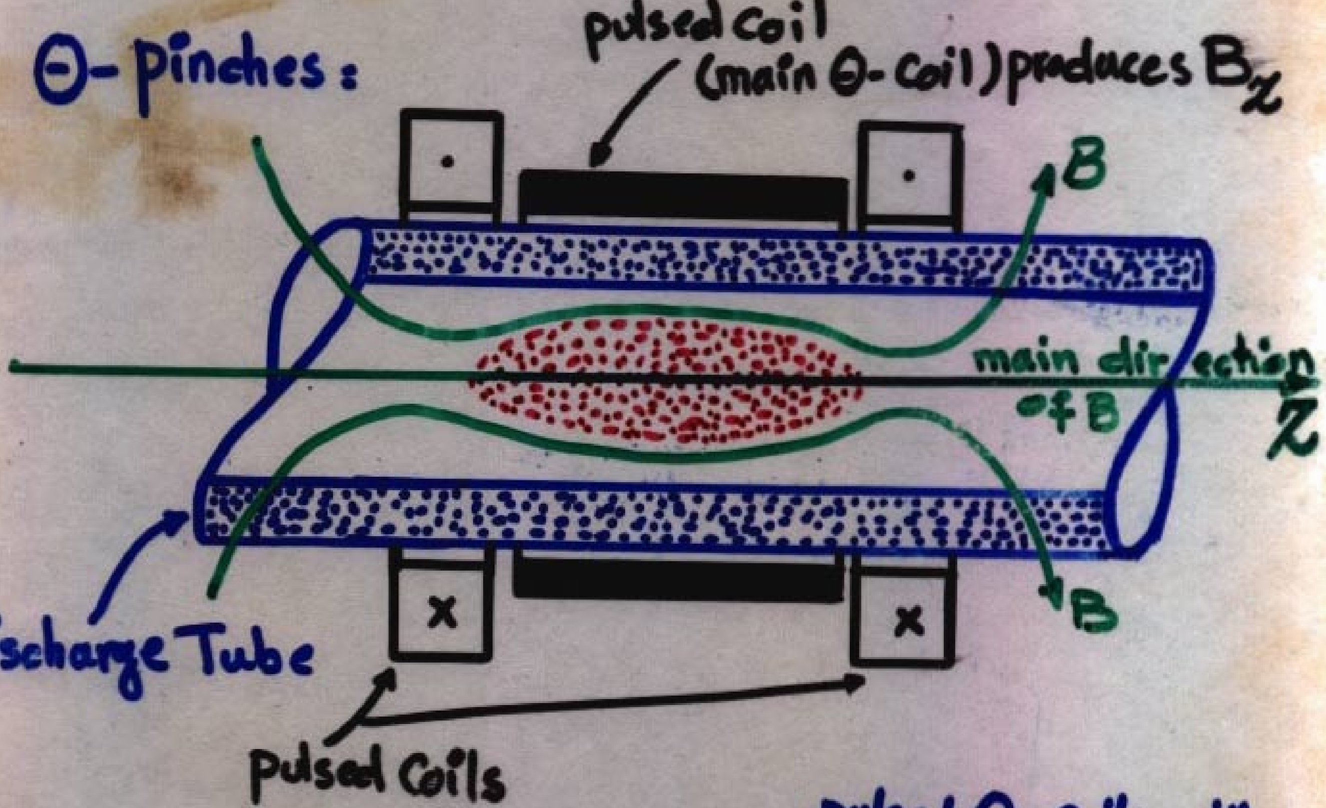
Then an additional pulsed current will be applied to the 1st half of the 2nd magnet.

- a further 3rd stage will be repeated, and in total, the kinetic energy is continuously increased through-out the 3-stages.

is adiabatic Compression applicable?

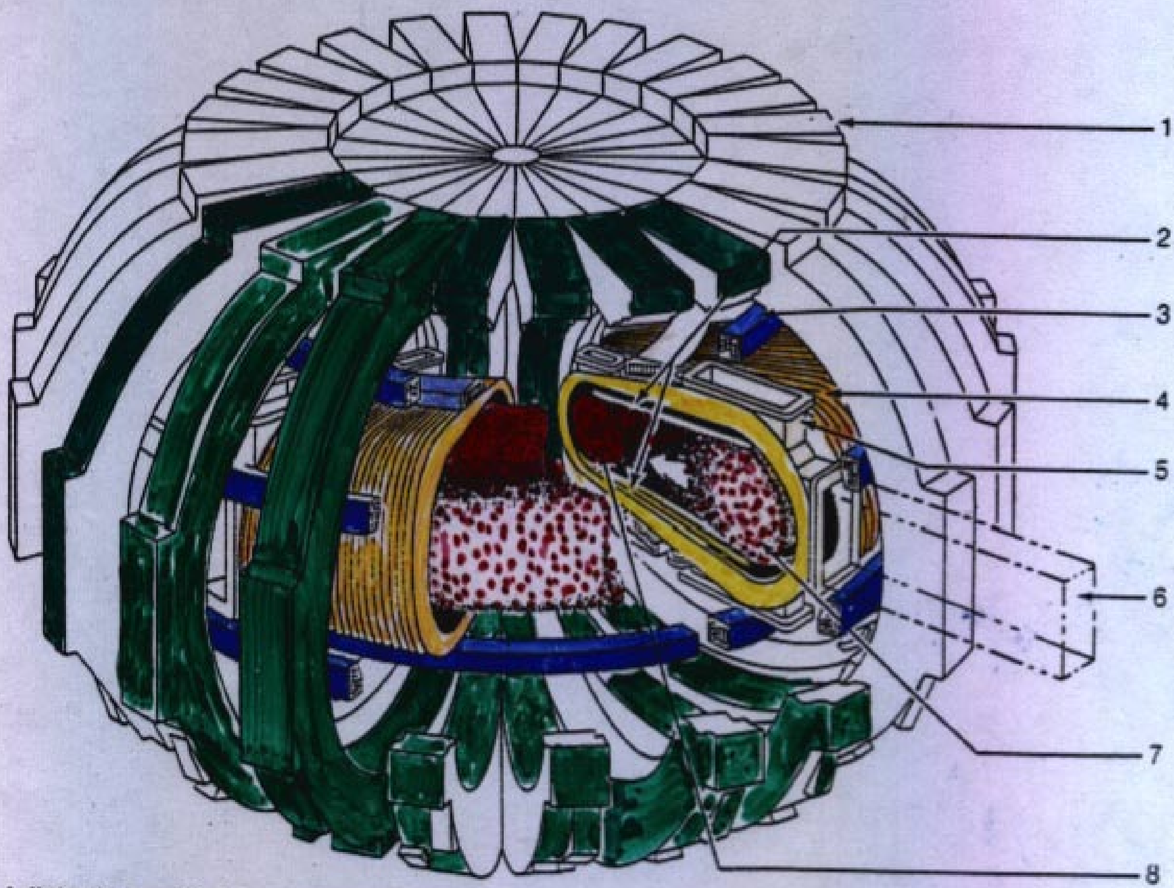
YES:

Θ- pinches:



$B_{high} > B_z$

- pulsed Θ- Coils will Produce the main B_z field, and also will Produce adiabatic compression
- B_{high} Coils will produce mirror + adiab. comp.



Adiabatic Toroidal Compresser (ATC)

1. Toroidal Field Coils (24)
2. Rail Limiters
3. Poloidal Field Coils
4. Corrugated Stainless Steel Vacuum Chamber
5. Port Cross (One of 6)
6. To Pumps (6)
7. Initial Ohmic-Heated Plasma
8. Compressed Plasma

In the Princeton ATC Tokamak, the plasma is adiabatically compressed (in both major and minor radii) to achieve a higher temperature than ohmic heating can provide. [Princeton University Plasma Physics Laboratory, sponsored by the U.S. Atomic Energy Commission.]