PLASMA LOSSES

- **Plasma Internal Energy Losses**
  - AS Most of the fuel is lost unreacted
- **Plasma Radiation Losses**
  - **Bremsstrahlung Radiation**
    - when charged particles are decelerated by Collision with other charged particles and emit a photon
  - **Radiative Recombination**
  - **Spectral line Radiation**
  - **Dielectronic Radiation**
  - **Cyclotron Radiation**
    - emitted from electrons when they gyrate in a strong magnetic field

Atomic Line Radiation

- result from a 3-body capture of a free electron by an ion with a third body in the vicinity. Binding energy of the electron is radiated.
- results if an impurity is raised to an excited state by electron impact.
- The capture of a free electron with simultaneous excitation of a second orbital electron, such that both electrons are in an excited state.

Plasma Internal Energy Losses

in a fusion reactor, only ~20% of the fuel is burned-up, Thus, most of the fuel is lost unreacted. The energies carried by the unburned fuel by charged reaction products and the external heating of the fuel.

The rate of internal energy loss :

$$P_E = \frac{3}{2} \sum_j \frac{n_j K T_j}{\tau_j}$$

τ_j = particle containment time

$$= \underbrace{\frac{3}{2} n_1 \frac{KT_1}{\tau_1} + \frac{3}{2} n_2 \frac{KT_2}{\tau_2}}_{\text{un-reacted fuel confined in the plasma}} + \underbrace{\frac{3}{2} n_{Pr} \frac{KT_{Pr}}{\tau_{Pr}}}_{\text{charged reaction Products confined in the plasma}} + \underbrace{\frac{3}{2} n_e \frac{KT_e}{\tau_e}}_{\text{electron population}}$$

Consider: $\tau_1, \tau_2, \tau_{pr}$ are long, so ionic species equilibrate to the same kinetic temperature

$$= \frac{3}{2} \frac{KT_i}{\tau_i} (n_1 + n_2 + n_{Pr}) + \frac{3}{2} \frac{KT_e}{\tau_e} n_e$$

For steady state operation "Quasi-neutral":

$$\tau_i \approx \tau_e \approx \tau_{p_r}$$

if the burn-up fraction is small,

i.e. $n_{p_r} \ll n_1$
$\ll n_2$

Then $n_i \approx n_1 + n_2$

if containment time is long enough for ions and electrons to equilibrate, i.e. $T_i \approx T_e$

So: $P_E = \frac{3}{2} \frac{KT}{\tau_{p_r}} (n_1 + n_2 + n_e)$

In a fully-ionized plasma with two fuel species:

$$n_e = Z_1 n_1 + Z_2 n_2$$

Z_1 and Z_2 are the proton # of fuel species

$$\frac{n_1}{n_i} + \frac{n_2}{n_i} = 1$$

$\frac{n_1}{n_i} = \gamma$ = fuel mixture ratio $\equiv \frac{\text{1st fuel species}}{\text{Total fuel}}$

$$\frac{n_1}{n_i} = \gamma, \quad \frac{n_2}{n_i} = 1 - \gamma$$

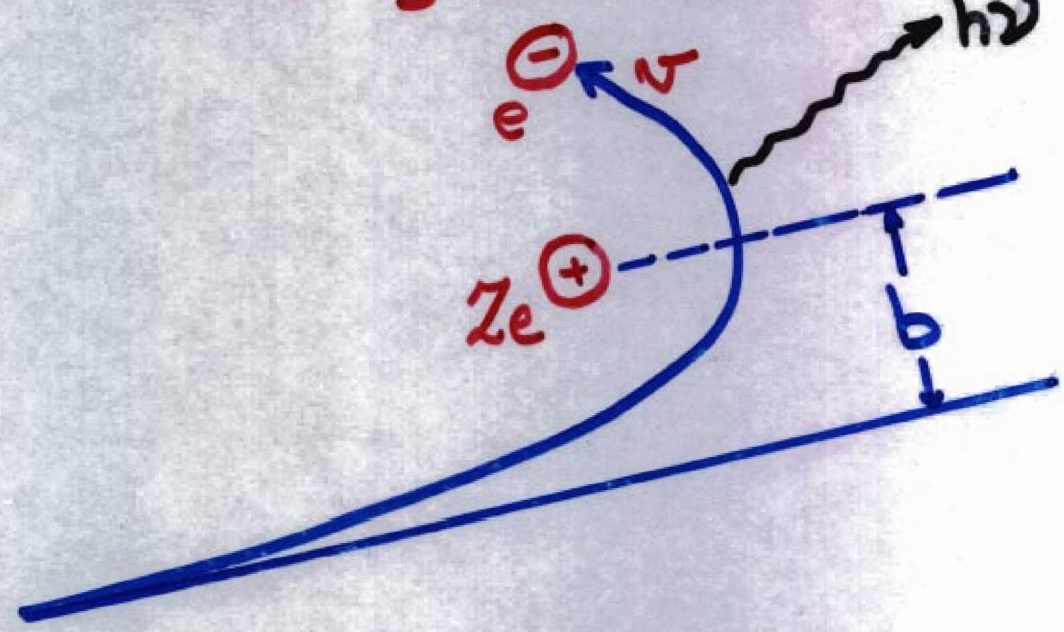
hence:

$$P_E = \frac{3}{2} \frac{KT}{\tau_{p_r}} \left[1 + \gamma Z_1 + (1-\gamma) Z_2 \right] n_i$$

w/m^3

Bremsstrahlung Radiation

Causes :- Energy loss
- Cooling mechanism



from classical Electromagnetic theory:
U; Total power radiated from a single electron of charge e, velocity $\bar{v}$, acceleration $\dot{v}$

$$\frac{dU}{dt} = \frac{e^2}{6\pi\epsilon_0 c^3} \frac{\left[|\dot{v}|^2 - (v \times \dot{v})^2/c^2\right]}{(1 - v^2/c^2)^3} \text{ watts}$$

non-relativistic limit: $v \ll c$

$$\frac{dU}{dt} = \frac{e^2 \dot{v}^2}{6\pi\epsilon_0 c^3} \text{ watts}$$

The Coulomb force $F_c = \dfrac{Ze^2}{4\pi\epsilon_0 b^2} = m_e \dot{v}$

hence: $\dot{v} = \dfrac{Z e^2}{4\pi \epsilon_0 m_e b^2}$

consequently:

$$\frac{dU}{dt} = \frac{Z^2 e^6}{96 \pi^3 \epsilon_0^3 c^3 m_e^2 b^4}$$

and $b = t_0 v_e$ || t_0: electron is in the vicinity of the ion for time t_0

The energy radiated / collision:

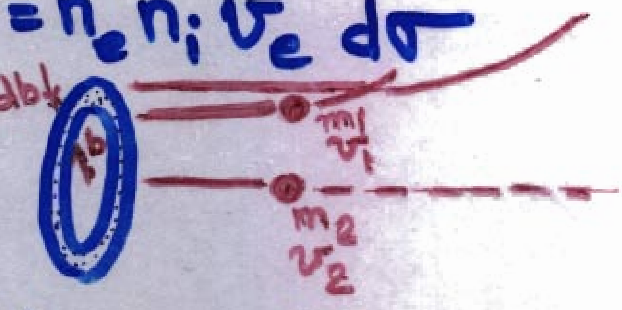
$$E_R \approx t_0 \frac{dU}{dt} = \frac{b}{v_e}\frac{dU}{dt} \quad \text{Joules}$$

TOTAL Radiated Power:

$$dP_{Br} = E_R \times r$$

r = reaction rate

$= n_e n_i v_e \, d\sigma$



$d\sigma = 2\pi b \, db$

$$= 2\pi n_e n_i \frac{dU}{dt} b^2 db$$

$$= \frac{Z^2 e^6 n_e n_i}{48 \pi^2 \epsilon_0^3 c^3 m_e^2} \frac{db}{b^2}$$

$$P_{Br} = \frac{Z^2 e^6 n_e n_i}{48\pi^2 \epsilon_0^3 C^3 m_e^2} \int_{b_{min}}^{b_{max} \to \infty} \frac{db}{b^2} \quad w/m^3$$

$$= \frac{Z^2 e^6 n_e n_i}{48\pi^2 \epsilon_0^3 C^3 m_e^2 \, b_{min}} \quad w/m^3$$

replace b_{min} by the Compton Wavelength for electrons: $b_{min} = \lambda_c = \frac{h}{2\pi m_e v_e}$

$$P_{Br} = \frac{n_e n_i Z^2 e^6 v_e}{24\pi \epsilon_0^3 C^3 m_e h}$$

cannot replace by λ_D as per the case for the rate of change of momentum, it is <u>NOT</u> a shielding case as a photon is emitted

for Maxwellian plasma

$$f(v) = \frac{dn_v}{dv}$$

$$= \frac{4n}{\pi^{1/2}} \left(\frac{m}{2KT}\right)^{3/2} v^2 \, Exp\left(-\frac{mv^2}{2KT}\right)$$

The mean thermal velocity:

"electrons": $\bar{v} = \frac{1}{n} \int_0^\infty v \, f(v) \, dv = \left(\frac{8KT_e}{\pi m_e}\right)^{1/2}$

$$P_{Br} = \frac{n_e n_i Z^2 e^6}{24\pi \epsilon_0^3 c^3 m_e h} \left(\frac{8KT_e}{\pi m_e}\right)^{1/2} \quad w/m^3$$

From Quantum mechanical calculations, the result is larger by a factor $\left(\frac{3}{2}\right)^{1/2}$

$$P_{Br}\Big|_{Q.M.} = \frac{e^6}{24\pi \epsilon_0^3 c^3 m_e h} \left(\frac{12KT_e}{\pi m_e}\right)^{1/2} n_e n_i Z^2$$

$$= 1.625 \times 10^{-38} n_e n_i Z^2 \underset{(ev)}{T_e}^{1/2}$$

* for two fuel species:

replace $n_i Z^2$ by $\sum_j n_j Z_j^2$

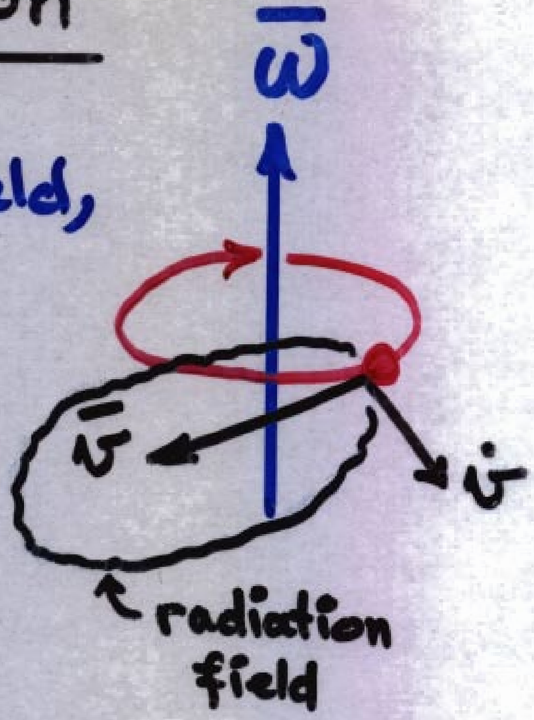
* for quasi-neutral plasmas:

replace n_e by $\sum_j n_j Z_j$

$$P_{Br} = 1.625 \times 10^{-38} \underset{(ev)}{T_e}^{1/2} \sum_j n_j^2 Z_j^3$$

for Advanced and Aneutronic fusion reaction; high Z-isotopes, which is a restriction. Bremsstrahlung is a Significant loss for DT and more Serious for higher Z fuels.

Cyclotron Radiation



in a strong magnetic field, an electron will gyrate with frequency ω. if electron is energetic, The acceleration will induce the emission of photons from the electron in the direction of it's velocity $\bar{v}$.

As per classical Electromagnetic theory:

$$\frac{dU}{dt} = \frac{e^2}{6\pi\epsilon_0 c^3} \frac{\left[|\dot{v}|^2 - (v \times \dot{v})^2/c^2\right]}{(1 - v^2/c^2)^3} \text{ watts}$$

The relativistic cyclotron frequency is:

$$\omega = \frac{eB}{m_e}\left(1 - \frac{v^2}{c^2}\right)^{1/2}$$

and the acceleration is: $\dot{v} = \omega v$

Since only $v_\perp$ contributes, as $\dot{v} \perp \omega$

hence:

$$\left[|\dot{v}|^2 - (v \times \dot{v})^2/c^2\right] =$$

$$= \omega^2 v_\perp^2 - \frac{\omega^2 v_\perp^4}{c^2} = \omega^2 v_\perp^2 \left(1 - \frac{v_\perp^2}{c^2}\right)$$

Substitute into dU/dt:

$$\frac{dU}{dt} = \frac{e^4 v_\perp^2 B^2}{6\pi\epsilon_0 c^3 m_e^2 \left(1 - v_\perp^2/c^2\right)} \text{ watts}$$

[for a single electron]

$$\left.\frac{dU}{dt}\right|_{\text{all electrons}} = n_e \frac{dU}{dt}$$

Consequently; for a Maxwellian plasma

$$P_{cyc} = \frac{e^4 B^2 n_e}{3\pi\epsilon_0 m_e^2 c} \left(\frac{KT_e}{m_e c^2}\right)\left[1 + \frac{5KT_e}{2 m_e c^2} + \cdots\right] \text{ w/m}^3$$

* N.B. integration is carried-out for an optically thin plasma

$$P_{cyc} = 6.21 \times 10^{-20}\, n_e \, T_{e\,(eV)} \, B^2 \left(1 + \frac{T_{e\,(eV)}}{2.04 \times 10^5} + \cdots\right)$$

w/m^3

The emitted photons from a fusion plasma have frequencies ~ 100 GHz, in the far infrared, and absorbed in the plasma. also, they are reflected from the metallic walls.

Radiation transport calculations showed that the plasma core radiates strongly so the plasma will be heated near the outer radius.

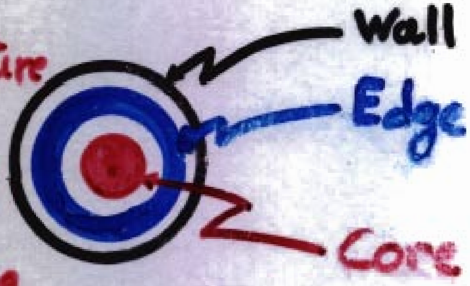
Thus: Causes:

* Cooling of the Core
* heating of outer periphery

Charge exchange, Ionization, Impurities and Line Radiation

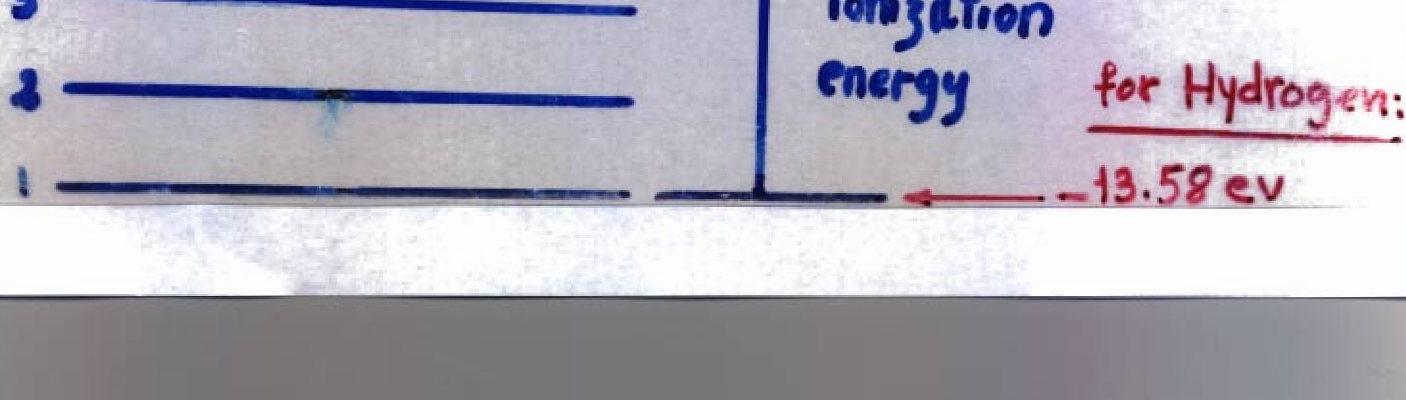
This is directly related to the Atomic Collision Phenomena. Atomic and Molecular Collisions are responsible for the charge exchange which will cause the lost of hot ions. The existance of impurities will absorb the electron energy through ionization, and consequently will emit line radiation which will be; mainly, absorbed by the fusion reactor walls.

INNER Confined plasma .... { • Core: High Temperature > 1Kev and density $\sim 10^{20}\,m^{-3}$
"Bremsstrahlung + Cyclotron radiation are ~~the~~ main, and most of the fusion will occur



Plasma outer layer .... { • Edge: lower temperature and The density might be the same or even higher, and atomic transitions are the main loss

Vacuum Vessel of The reactor ... { • Wall: will absorb line radiation AND This might cause wall damage



- $e^- + H \longrightarrow H^* + e^-$ — excitation

neutral atom (H) — excited atom (H^*)

That's inelastic scattering if

$e^- + H \longrightarrow e^- + H$

That's elastic

Takes place when the electron populates one of the excited states, i.e. the atom will be overpopulated by an extra electron.

- $H^* \longrightarrow H + h\nu$ — de-excitation

The excited atom is de-excited to the ground state and the ionization energy is released in the form of a photon.

- $$\left.\begin{array}{l} e^- + H \longrightarrow H^+ + e^- + e^- \\ e^- + H_2 \longrightarrow H_2^+ + e^- + e^- \end{array}\right\}$$ — ionization of an atom or a molecule

ion (H^+)

molecule (H_2)

This takes place when the electron transfers enough energy to the bound electron, so as the later will leave the atom (or molecule) and becomes free.

- $e^- + H^+ \longrightarrow H + h\nu$ — recombination "radiative"

The ion will attract a free electron to the ground state and RECOMBINE to form a neutral atom, and the binding energy will be released in the form of a photon "radiation".

- $e^- + H_2^+ \longrightarrow \underline{H} + \underline{H}^+ + e^-$ — $\boxed{\text{dissociation}}$

molecular ion "ionic molecule" (H_2^+); energetic atom ($\underline{H}$); energetic ion ($\underline{H}^+$)

in this case; the formation of an energetic atome and an energetic ion will take place, which means that an atom will be formed, and if:-

$e^- + H_2^+ \longrightarrow H^* + H^*$ — $\boxed{\text{Recombination "Dissociative"}}$

excited atom (H^*)

excited atoms are formed, the dissociation is called DISSOCIATED RECOMBINATION

- $e^- + O^{++} \longrightarrow O^{+**}$ doubly excited oxygen ion

$\longrightarrow O^+ + h\nu_1 + h\nu_2$ — $\boxed{\text{Recombination "dielectronic"}}$

The free electron is captured + Simultaneous excitation, which results in the formation of an ion and the release of photons.

- $e^- + O^{++} \longrightarrow O^{+**}$

$\longrightarrow O^{++} + e^-$ — $\boxed{\text{autoionization}}$

This will take place when one of the excited electrons acquires enough energy to leave the atom.

- $H + \underline{H}^+ \longrightarrow \underline{H} + H^+$ — $\boxed{\text{charge exchange}}$

excited ion ($\underline{H}^+$); excited atom ($\underline{H}$)

usually occurs for ions and atoms of like species.

EQUILIBRIUM

Consider the atom density in an excited state 1 to be n_1, in state 2 to be n_2; and consider W_1 and W_2 to be the corresponding energy states. According to Maxwell-Boltzmann distribution for excited states:

$$\frac{n_1}{n_2} = \frac{g_1}{g_2} \exp\left\{\frac{-(\Delta W)}{KT}\right\}$$

$\Delta W = W_1 - W_2 =$ ionization Potential

g: is the degeneracy factor

For the case of high-density Plasmas: state 2 : neutral atom; state 1 : electron-ion pair

then: $$\frac{g_1}{g_2} = \left(\frac{2\pi m_e KT}{h^2}\right)^{3/2} \cdot \frac{1}{n_1}$$

hence:

$$\frac{n_i^2}{n_2} = \left(\frac{2\pi m_e KT}{h^2}\right)^{3/2} \exp\frac{-\Delta W}{KT}$$

n_i: ion density; n_2: neutral atom density

hence:

$$\frac{n_i}{n_n} = 3.022 \times 10^{27} \frac{T_{ev}^{3/2}}{n_i} \exp\left(-\frac{\Delta W}{T}\right)$$

SAHA equation

The equation tells the expected amount of ionization in a gas in thermal equilibrium

The fractional ionization is given by $\frac{n_i}{n_i + n_n}$

$\approx \frac{n_i}{n_n}$ $\quad \| \; n_i \ll n_n$

To understand the meaning we can consider Air at room temperature of 300°K (0.026 ev) with $\Delta W = 14.5$ ev for nitrogen, and n_n of the value $3 \times 10^{25}\ m^{-3}$.

This gives a fractional ionization:

$$\frac{n_i}{n_n} \approx 10^{-122}$$

with temp. increase, the degree of ionization remains low until ΔW becomes few times T, then $\frac{n_i}{n_n}$ rises abruptly and the gas becomes in a plasma state. With further increase in T, n_n becomes $< n_i$ and the plasma becomes FULLY IONIZED

in high-density plasmas, Collisional processes may dominate de-excitation and recombination and the relative populations in Thermodynamic equilibrium are governed by the Saha eqn., depending only on Temperature.

if the total heavy particle density ($n_t = n_i + n_n$) is introduced to the Saha eqn., then:

$$\frac{n_i}{n_t} \approx 1 - \left[3.313 \times 10^{-28} n_t T_{ev}^{-3/2} e^{\left(\frac{\Delta W}{T}\right)} \right]$$

let us consider hydrogen gas at T= 10 ev at a pressure of 5×10^{-3} atm.

$$n_t = \frac{P}{\underbrace{KT}_{\text{Joules}}}$$

P in Pascal = newton/m^2
1 atm = 1.013×10^5 Pa
1 Torr = 133.3 Pa

$P = 5 \times 10^{-3} \times 1.013 \times 10^5 = 506.5$ Pa

$KT = 10 \times 11609 \times 1.38 \times 10^{-23} = 1.6 \times 10^{-18}$

$n_t = 3.166 \times 10^{20}$ m^{-3}

$$\frac{n_i}{n_t} = 1 - \left[3.313 \times 10^{-28} \times 3.166 \times 10^{20} \times (10)^{-3/2} e^{\frac{13.6}{10}} \right]$$

$$= 1 - 1.29 \times 10^{-5} \approx 1 \quad \text{i.e. } n_i \approx n_t$$

100% ionization ⇒ Fully ionized

if T = 1 ev ; $n_t = 3.166 \times 10^{21}$ m^{-3}

and $\frac{n_i}{n_t} = 84.5\%$ ionization

if T = 0.1 ev ⟶ ≈ 0% ionization ($1 - 3.8 \times 10^{52}$)

To the solar system:

(hot plasma with low density)

there is a balance between:-

[rate of Collisional excitation
rate of ionization]

AND

[rate of radiative de-excitation
rate of radiative recombination]

} Determines the Populations of excited levels

$\underbrace{\qquad\qquad\qquad}$

CORONAL EQUILIBRIUM

i.e. depends on the various rate of coeff. $\langle \sigma v \rangle$

For a hot plasma (i.e. high temp.), at the Core:

rate of recombination $= n_e n_i \langle \sigma_r v_e \rangle$

rate of ionization by electrons $= n_e n_n \langle \sigma_e v_e \rangle$

rate of ionization by ions $= n_i n_n \langle \sigma_i v_i \rangle$

[$n_n \rightarrow$ neutral density]

hence:

$$n_e n_i \langle \sigma_r v_e \rangle = n_e n_n \langle \sigma_e v_e \rangle + n_i n_n \langle \sigma_i v_i \rangle$$

at the core: neglect the rate of collisional excitation as well as the rate of radiative de-excit$\underline{n}$.

hot-low density plasma: $n_e \approx n_i$

$$\frac{n_i}{n_n} = \frac{\langle \sigma_e v_e \rangle + \langle \sigma_i v_i \rangle}{\langle \sigma_r v_e \rangle}$$

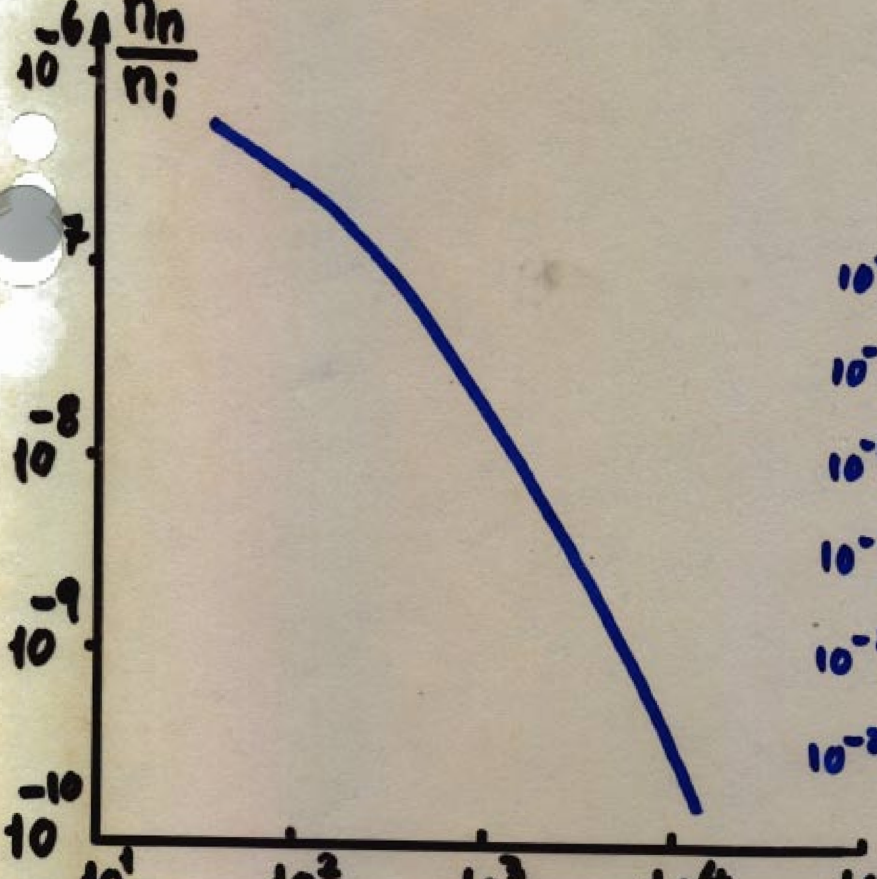
and the fractional concentration of neutral Atoms $= \dfrac{n_n}{n_i + n_2}$

$$\frac{n_n}{n_i+n_n} = \frac{\langle \sigma_r v_e \rangle}{\langle \sigma_r v_e \rangle + \langle \sigma_e v_e \rangle + \langle \sigma_i v_i \rangle}$$

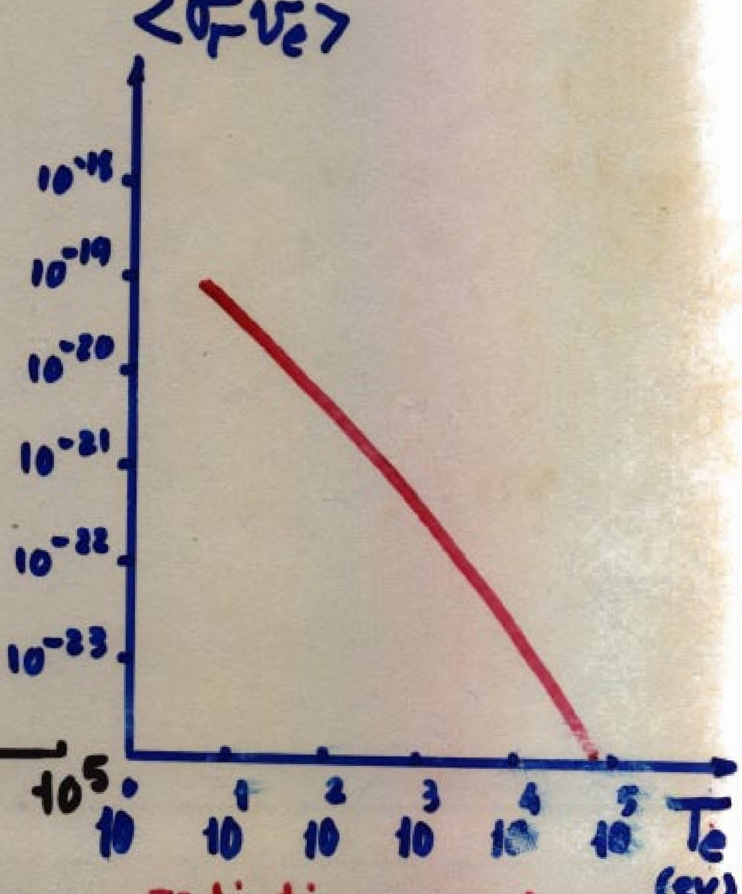
$$\cong \left[\frac{1}{1 + \dfrac{\langle \sigma_e v_e \rangle}{\langle \sigma_r v_e \rangle}} \right]$$

$\langle \sigma_i v_i \rangle$ very small, could be neglected

i.e. depends on the temp. as $\langle \sigma_e v_e \rangle$ and $\langle \sigma_r v_e \rangle$ are functions of temp.



ratio of neutral atom density to plasma density



radiative recombination rate coefficient for Hydrogen

I.2 PLASMA RADIATION LOSSES

Bremsstrahlung Radiation

Deceleration of Charged particles by Collission with Other Charged particles

photon
$h\nu$

Cyclotron Radiation

emission of photons due to electron gyration in strong magnetic fields

Atomic Line Radiation

- Radiative recombination
- Spectral line radiation
- Dielectronic radiation

Bremsstrahlung : $\nearrow$ Energy Loss $\searrow$ Cooling Mechanism

$$P_{Br} = \left(\frac{e^6}{24\pi \epsilon_0^3 C^3 m_e h}\right)\left(\frac{12KT_e}{\pi m_e}\right)^{1/2} n_e n_i Z^2$$

Two fuel species: $n_i Z^2 \Rightarrow \sum_j n_j Z_j^2$

Quazi-neutral plasmas: $n_e \Rightarrow \sum n_j Z_j$

FINAL

$\Downarrow$

$$\underset{(W/m^3)}{P_{Br}} = 1.625 * 10^{-38} \underset{(eV)}{T_e^{1/2}} \sum_j n_j^2 Z_j^3$$

$$\cong 5.35 * 10^{-37} \underset{(m^{-3})(m^{-3})}{n_e n_i} Z^2 \underset{(KeV)}{T_e^{1/2}}$$

$$\underbrace{\Delta P_{Br}}_{\text{ratio with impurities}} = 1 + \gamma (Z + Z^2) + \gamma^2 Z^3$$

γ → impurity fraction; Z → impurity Z.

Cyclotron:

→ Core Cooling
→ outer periphery heating

$$P_{cyc} = \frac{e^4 B^2 n_e}{3\pi \epsilon_0 m_e^2 C}\left(\frac{K T_e}{m_e C^2}\right)\left(1 + \frac{5 K T_e}{2 m_e C^2} + \dots\right)$$

$$\underset{(w/m^3)}{P_{cyc}} \approx 6.21 * 10^{-17} \underset{(m^{-3})}{n_e} \underset{(Kev)}{T_e} \underset{(Tesla)}{B^2} \left(1 + \frac{T_e^{(Kev)}}{146}\right)$$

Atomic Line:

- Spectral line radiation:

$$\underset{(W/m^3)}{P_{LR}} \approx 8 * 10^{-35} \underset{(m^{-3})}{n_e} \underset{(m^{-3})}{n_i} Z^6 \underset{(Kev)}{T_e^{3/2}}$$

Dominant factor at lower plasma temperature

- Radiative Recombination:

$$\underset{(W/m^3)}{P_{RC}} \approx 6 * 10^{-40} \underset{(m^{-3})}{n_e} \underset{(m^{-3})}{n_i} Z^4 \underset{(Kev)}{T_e^{1/2}}$$

effective at higher temp.

The NON-radiative losses belong to the branch of INTERNAL ENERGY LOSSES due to the fact that only 20% of the fuel will be burned, the remaining un-burned fuel is then Lost unreacted.

$$\underset{(W/m^3)}{P_E} \approx 2.44 * 10^{-16} \left[1 + \gamma Z_1 + (1-\gamma) Z_2\right] \frac{n_i T}{\tau_{Pr}}$$

fuel ratio

$n_i \Rightarrow m^{-3}$

$T \Rightarrow KeV$

$\tau_{Pr} \Rightarrow sec \approx \tau_i \approx \tau_e$ for Steady State operation

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Low plasma temperature: P_{LR} is dominant

high plasma temperature: P_{RC} & P_{BR}

Very high plasma temperature: P_{BR}

Total Radiation Power Loss

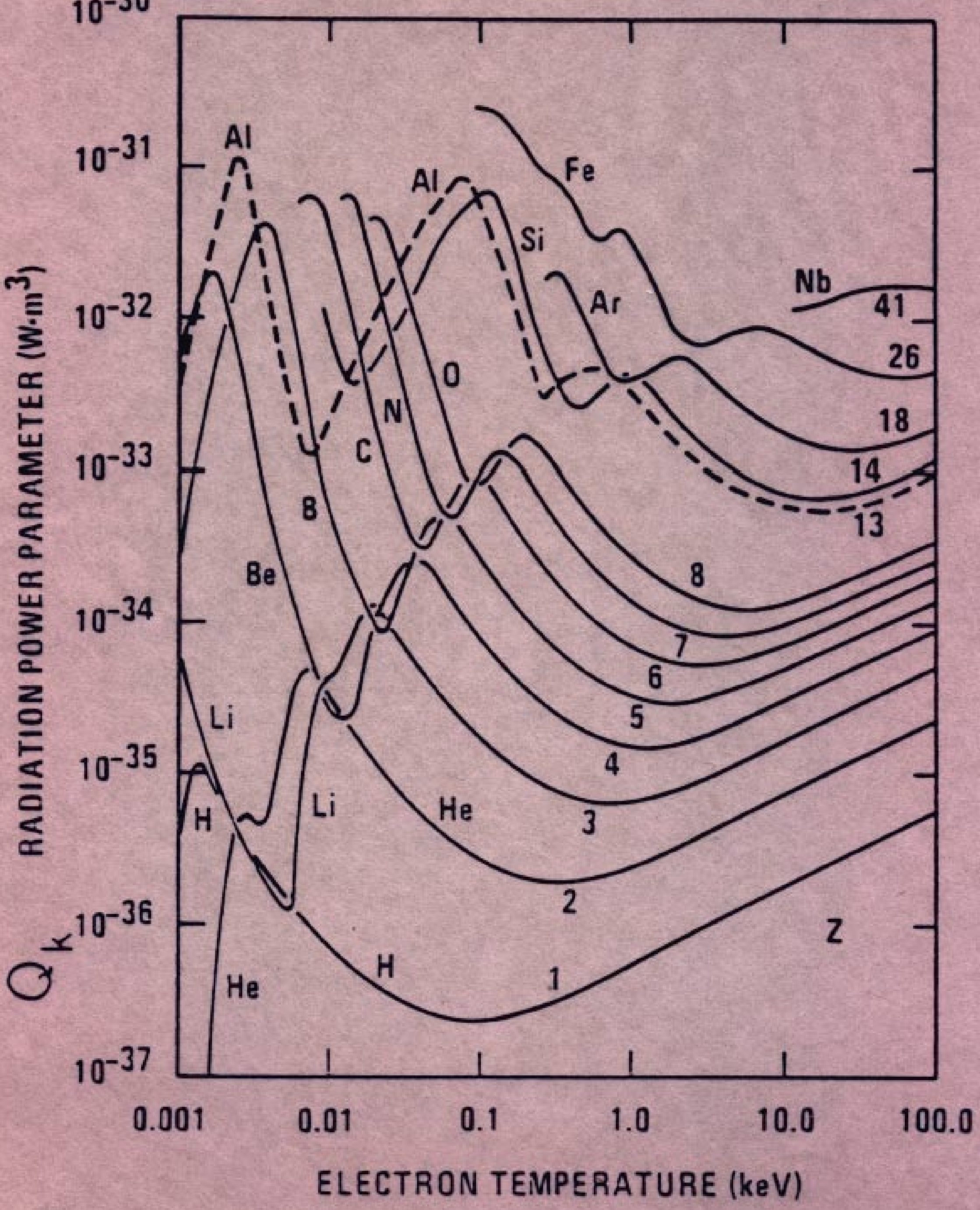
$$= \underbrace{P_{BR} + P_{LR} + P_{RC}}_{\text{function of } n_e \text{ and } n_{impurities} \Downarrow (n_K)} + \underbrace{K_{cyc}}_{\text{fraction absorbed in the walls}} P_{cyc}$$

$\therefore$ Radiation Power Parameter Q_K (W/m³) =

$$= \frac{P_{BR} + P_{LR} + P_{RC}}{n_e n_K}$$

i.e.

$$P_{RAD} = \underbrace{\sum_K n_e n_K Q_K}_{\text{from tables and curves, or Calculate and SUM-UP}} + \underbrace{K_{cyc} P_{cyc}}_{\text{To be calculated separately}}$$



The radiation power parameter as a function of electron temperature for various elements. (Cyclotron radiation losses must be computed separately.) From G. R. Hopkins and J. M. Rawls, Nuclear Technology 43, 332 (1979), Fig. 1.

To Calculate K_{cyc}:

$$K_{cyc} = f(T_e, D)$$

K_{cyc}: available in tables and curves for cylindrical plasmas

D: plasma depth

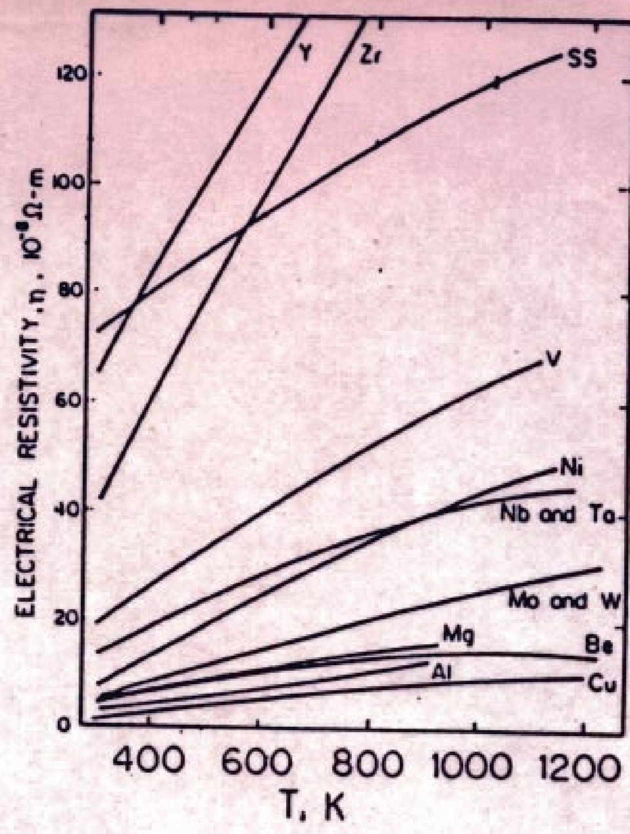
$$D = \frac{2 n_e a}{B^{3/2} \eta^{1/2} (1-\beta)^{3/4}}$$

$a \equiv$ plasma radius (m)

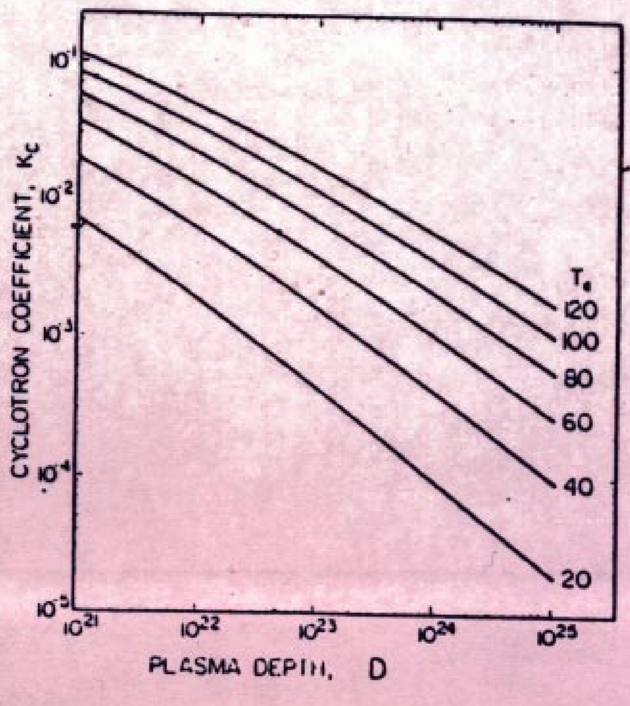
$\eta \equiv$ material resistivity ($\Omega . m$)

$$\beta = \frac{P}{B^2/2\mu_0} = \frac{\text{plasma Kinetic Pressure}}{\text{magnetic pressure}}$$

steps:
- get η at the wall temperature
- get β from $P = \sum_j n_j K T_j$ & B
- get D
- use the curves.



Electrical resistivity vs. temperature for various metals. From G. H. Miley, Fusion Energy Conversion, American Nuclear Society, LaGrange Park, IL, 1975.



Cyclotron radiation coefficient K_C vs. "plasma depth" D, for a cylindrical plasma with various T_e (keV).