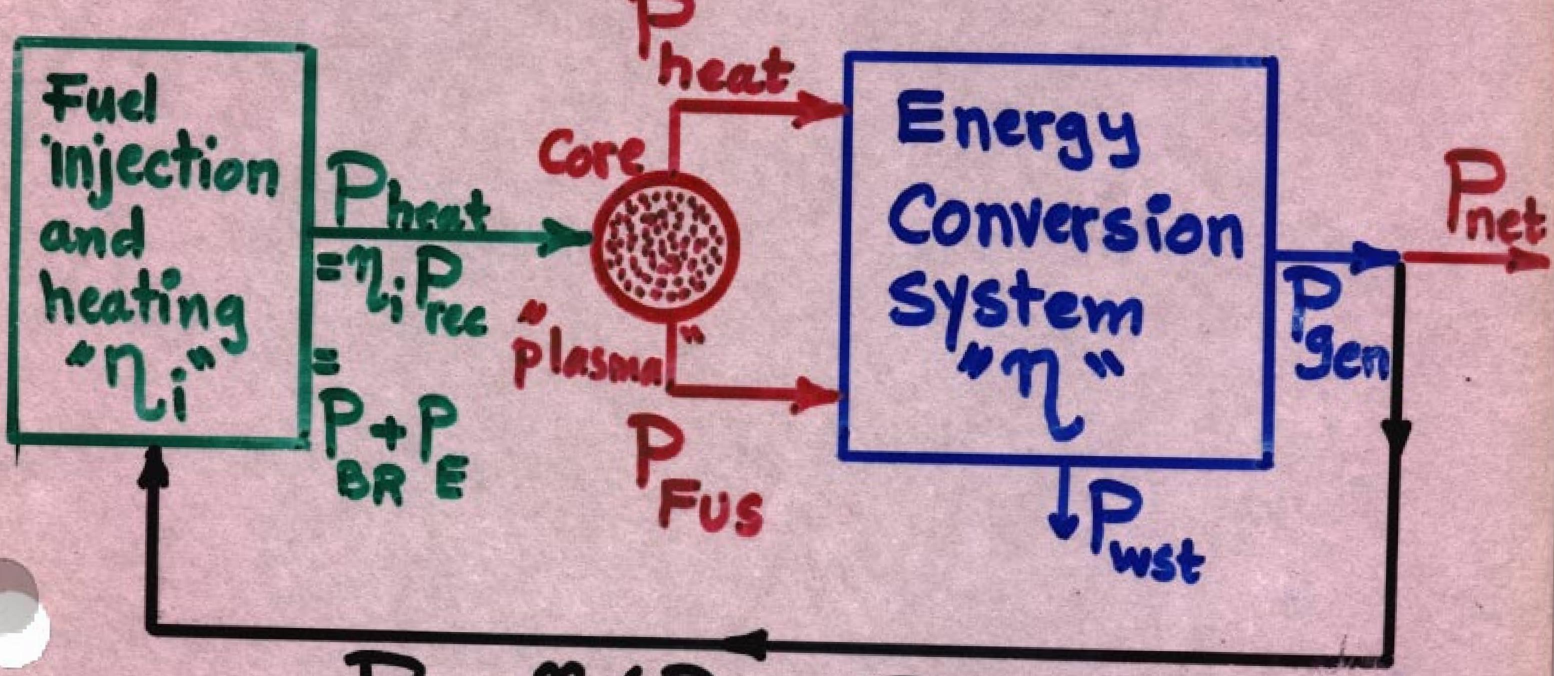
I.3 Fusion Reactor Power Balance



$$P_{rec} = \eta\,(P_{heat} + P_{Fus}) - P_{net}$$

$$P_{wst} = \frac{1-\eta}{\eta} P_{gen}$$

$$\underbrace{P_{gen}}_{\text{Electrical}} = \eta\,(\underbrace{P_{heat} + P_{Fus}}_{\text{Thermal}})$$

$$= \eta\,(P_{BR} + P_E + P_{Fus})$$

$$= \eta\,(P_{gen} + P_{wst})$$

$$P_{rec} = P_{gen} - P_{net}$$

$P_{heat} \Rightarrow$ for thermal heating of the ions

$$P_{heat} = \eta_i P_{rec}$$

$$= \eta_i \left[\eta \left(P_{BR} + P_E + P_{Fus} \right) - P_{net} \right]$$

$$= P_{BR} + P_E$$

$$\frac{P_{rec}}{P_{gen}} = \underbrace{F}_{\text{recirculating Power fraction}} = \frac{P_{heat}}{\eta_i P_{gen}}$$

F	> 1	= 0	= 1	< 1
POWER	External needed	NO recirc-ulation	NO net Power	NET POWER
CLASS-IFICATION	Driven reactor	Self-Sustained	Lawson reactor	Producing reactor

$$F\eta\eta_i = \frac{P_{heat}}{P_{heat} + P_{FUS}}$$

$$= \frac{\text{Plasma Heating Power}}{\text{Thermal power}}$$

$$\equiv \text{Heating Power Fraction}$$

$$\therefore Q = \frac{1}{F\eta\eta_i}$$

η and η_i are less than UNITY
(You cannot get 100% efficiency in any system)

"Re-Classify according to F possibilities"

GET: $Q \rightarrow$ Driven
$\rightarrow$ Self-Sustained
$\rightarrow$ Lawson
$\rightarrow$ Producing

I.4 LAWSON CRITERIA

Based on the heating power fraction for fusion reactors classification, one may substitute for the P_{heat} and P_{FUS} in order to extract the operational limits.

$$F\eta\eta_i = \frac{P_{heat}}{P_{heat} + P_{FUS}}$$

$$P_{heat} = P_{BR} + P_E$$

<u>for P_{FUS}:</u>

reaction rate / $m^3 = r = n_1 n_2 \langle \sigma v \rangle_{12}$

$\therefore$ Energy deposition rate $(W/m^3) = P_c =$

$= r E_c$ ...... where E_c = energy of charged reaction product/reaction

$$= E_c n_1 n_2 \langle \sigma v \rangle_{12}$$

at Equilibrium: $n_i \approx n_1 + n_2$

i.e. $n_1 = \gamma n_i$, $n_2 = (1-\gamma) n_i$

and hence: γ = fuel ratio

$$P = E n_i^2 \underbrace{\gamma(1-\gamma)}_{g_{12}} \langle \sigma v \rangle_{12}$$

[and $\gamma = 1/2$ for like-species]

if The blanket has a multiplication factor; then the Blanket Multiplication Factor $M = \ldots$

$$\ldots = \frac{\text{Total Energy / Fusion event}}{\text{Energy / Fusion event}}$$

Then:

$$P_{FUS} = M g_{12} E_{12} \langle \sigma v \rangle_{12} \, n_i^2$$

Substituting P_{FUS}, P_{BR}, P_E in the

$F\eta\eta_i$ equation:

$$Q = \frac{1}{F\eta\eta_i} = 1 + \frac{P_{FUS}}{P_{BR} + P_E}$$

$$P_{FUS} = ME_{12}\, g_{12} \langle\sigma v\rangle_{12}\, n_1^2$$

$$P_E \cong 2.44 * 10^{-16} \left[1 + \gamma Z_1 + (1-\gamma) Z_2\right] \frac{n_i T}{\tau}$$

$$P_{BR} \cong 5.35 * 10^{-37}\, n_e n_i Z^2\, T_e^{1/2}$$

$T_e \approx T_i$

$$n_e = \sum_j n_j Z_j \quad \text{"fully ionized"}$$
$$= n_i \left[\gamma Z_1 + (1-\gamma) Z_2\right]$$

$$n_i Z^2 = \sum_j n_j Z_j^2$$
$$= n_i \left[\gamma Z_1^2 + (1-\gamma) Z_2^2\right]$$

Hence:

$$P_{BR} \cong 5.35 * 10^{-37}\, n_i^2 \left[\gamma Z_1 + (1-\gamma) Z_2\right] *$$
$$* \left[\gamma Z_1^2 + (1-\gamma) Z_2^2\right] T^{1/2}$$

Substitute, and Solve for $n_i \tau$:

$$n_i \tau = \frac{2.44 * 10^{-16}\, T \left[1 + \gamma Z_1 + (1-\gamma) Z_2\right] \left(\frac{1}{F\eta\eta_i} - 1\right)}{M g_{12} E_{12} \langle \sigma v \rangle_{12} - 5.35 * 10^{-37}\, T^{1/2} \left[\gamma Z_1 + (1-\gamma) Z_2\right] * \left[\gamma Z_1^2 + (1-\gamma) Z_2^2\right] \left(\frac{1}{F\eta\eta_i} - 1\right)}$$

$n_i \tau \equiv$ LAWSON PARAMETER

FOR Hydrogenic plasmas : $Z_1 = Z_2 = 1$

$$n_i \tau \Big)_{\text{Hydrogenic}} = \frac{4.88 * 10^{-16}\, T \left(\frac{1}{F\eta\eta_i} - 1\right)}{M E_{12} g_{12} \langle \sigma v \rangle_{12} - 5.35 * 10^{-37}\, T^{1/2} \left(\frac{1}{F\eta\eta_i} - 1\right)}$$

FOR $M = 1$, $\gamma = 50\%$: $g_{12} \Rightarrow \frac{1}{4}$

$$n_i \tau = \frac{19.5 * 10^{-16}\, T \left(\frac{1}{F\eta\eta_i} - 1\right)}{E_{12} \langle \sigma v \rangle_{12} - 21.4 * 10^{-37}\, T^{1/2} \left(\frac{1}{F\eta\eta_i} - 1\right)}$$

The Heating Power Fraction Limits

one can characterize the power flow in an idealized fusion reactor by the heating power fraction $F\eta\eta_i$.

The Lawson parameter $n\tau$ is alway +ve and ranges over $0 \leq n\tau \leq \infty$, and Consequently this range could be used to find the limits of $F\eta\eta_i$.

$$n\tau = \frac{2.44 \times 10^{-16}\, T\,[1+\gamma Z_1+(1-\gamma)Z_2]\left[\frac{1}{F\eta\eta_i}-1\right]}{M E_{12}\, g_{12} \langle \sigma v \rangle_{12} - 5.35 \times 10^{-37} * T^{1/2}[\gamma Z_1+(1-\gamma)Z_2] * [\ldots \gamma Z_1^2+(1-\gamma)Z_2^2] * \left[\frac{1}{F\eta\eta_i}-1\right]}$$

hence: $n\tau = 0$ when numerator $= 0$

$= \infty$ when denominator $= 0$

So: for $n\tau = 0$; $\frac{1}{F\eta\eta_i} - 1 = 0$, i.e. $\underbrace{F\eta\eta_i}_{max.} = 1$

i.e. the plasma heating power = gross thermal power

"NO energy conversion system can provide that"

AND $n\tau = 0$ means that NO FUSION REACTION WILL TAKE PLACE

As $\eta < 1$, hence: NEEDED External Energy & NO P_{FUS}.

For $n\tau = \infty$:

$$M g_{12} E_{12} \langle \sigma v \rangle_{12} = 5.35 * 10^{-37} * T^{1/2} * [\gamma Z_1 + (1-\gamma) Z_2] * \cdots$$

$$\cdots [\gamma Z_1^2 + (1-\gamma) Z_2^2] \left[\frac{1}{F \eta \eta_i} - 1 \right]$$

from which:

$$F \eta \eta_i = \frac{P_{BR}}{\underbrace{M E_{12} g_{12} \langle \sigma v \rangle_{12}}_{P_{FUS}} + P_{BR}}$$

As $P_{BR} \ll P_{FUS}$

So: $$\left. F \eta \eta_i \right)_{min} \approx \frac{P_{BR}}{M E_{12} g_{12} \langle \sigma v \rangle_{12}}$$

$$\approx \frac{5.35 * 10^{-37} * T^{1/2} [\gamma Z_1 + (1-\gamma) Z_2][\gamma Z_1^2 + (1-\gamma) Z_2^2]}{M E_{12} g_{12} \langle \sigma v \rangle_{12}}$$

$$\approx \frac{5.35 * 10^{-37} T^{1/2} [\gamma^2 Z_1^3 + (1-\gamma)^2 Z_2^3 + \gamma(1-\gamma) Z_1 Z_2 (Z_1 + Z_2)]}{M E_{12} g_{12} \langle \sigma v \rangle_{12}}$$

i.e. $F \eta \eta_i \propto Z^3$ "OF The highest Z of the used fuel"

i.e. smaller $\left. F \eta \eta_i \right)_{min}$ for Hydrogenic plasmas

higher $\left. F \eta \eta_i \right)_{min}$ for Advanced and Aneutronic reactions

?? ECONOMY ??

From the previous:

$$F\eta\eta_i)_{min} \leq \underbrace{F\eta\eta_i \leq 1}$$

An Idealized Fusion Reactor
MUST operate within this region.

For unlike fuel and $\gamma = 0.5$; and $M = 1$

$$\gamma^2 Z_1^3 + (1-\gamma)^2 Z_2^3 + \gamma(1-\gamma) Z_1 Z_2 (Z_1 + Z_2) =$$

$$= 0.5^2 \left[Z_1^3 + Z_2^3 + Z_1 Z_2 (Z_1 + Z_2) \right]$$

and $g_{12} = \gamma(1-\gamma) = 0.5^2$

hence: $$F\eta\eta_i)_{min} \approx \frac{T^{1/2} * 5.35 * 10^{-37} \left[Z_1^3 + Z_2^3 + Z_1 Z_2 (Z_1 + Z_2) \right]}{E_{12} \langle \sigma v \rangle_{12}}$$

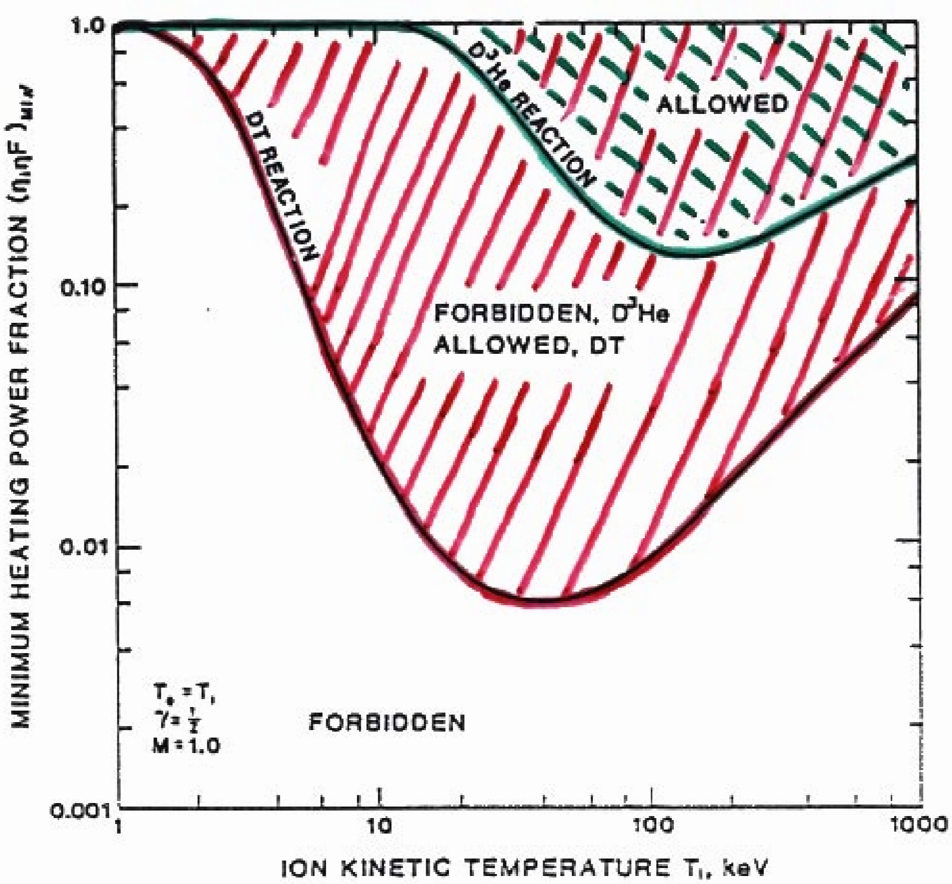
FOR Hydrogenic plasmas: D-T

$$F\eta\eta_i)_{min} \approx \frac{2.14 * 10^{-36}\ T^{1/2}}{E_{DT} \langle \sigma v \rangle_{DT}}$$

FOR Hydrogenic plasmas: D-D:

$g_{12} = \frac{1}{2}$

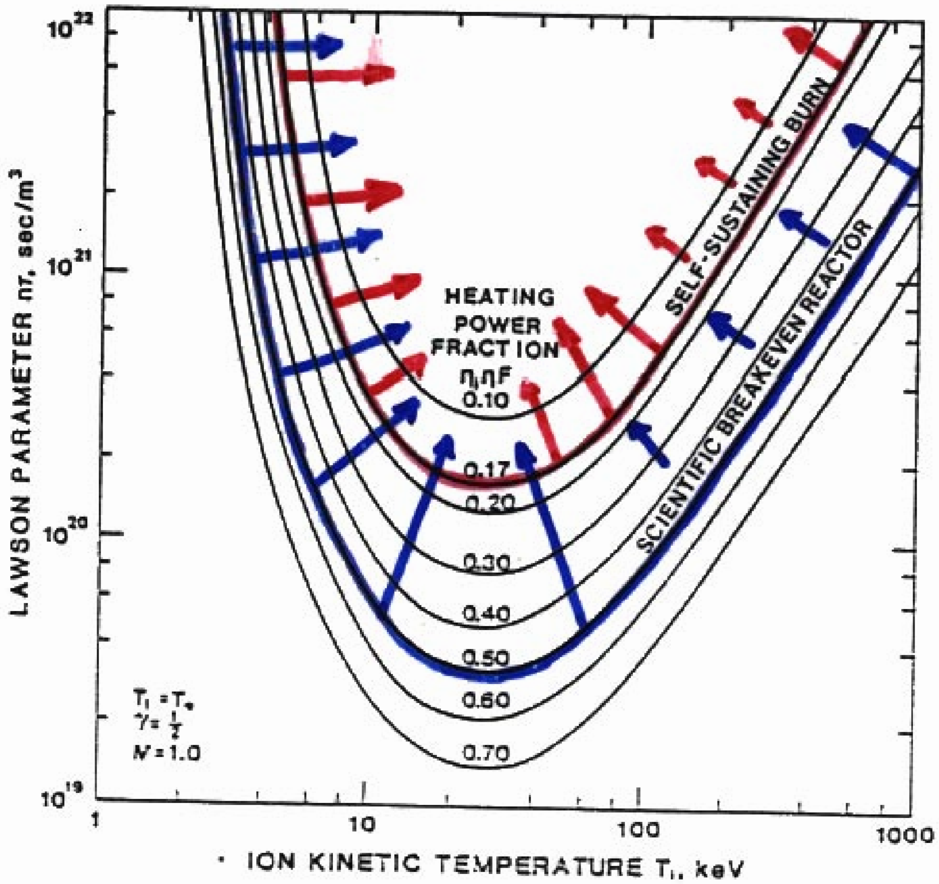
$$F\eta\eta_i)_{min} \approx \frac{1.07 * 10^{-36}\ T^{1/2}}{E_{DD} \langle \sigma v \rangle_{DD}}$$



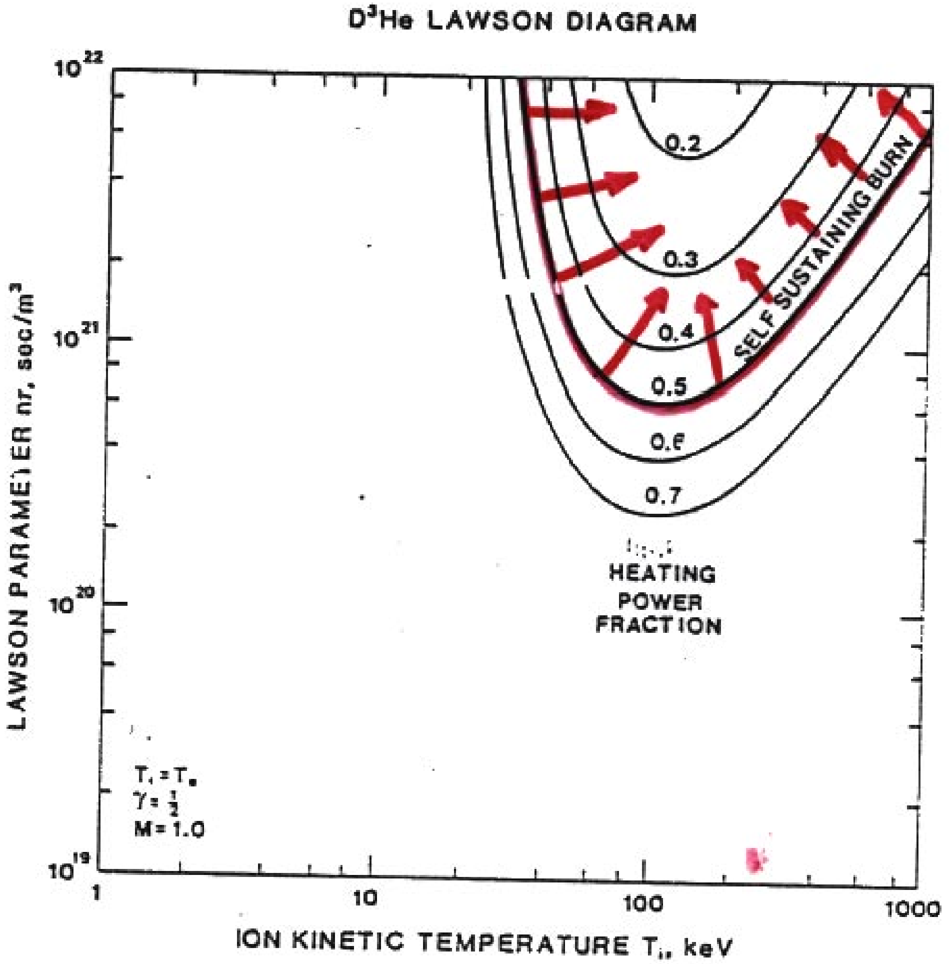
$$\left.(F\eta\eta_i)_{min}\right|_{DT} = \frac{1.07 * 10^{-36}\ T^{1/2}}{E_{DT}\langle\sigma v\rangle_{DT} + 1.07 * 10^{-36}\ T_{Kev}^{1/2}}$$

$$\left.(F\eta\eta_i)_{min}\right|_{D^3He} = \frac{3.21 * 10^{-35}\ T^{1/2}}{E_{D^3He}\langle\sigma v\rangle_{D^3He} + 3.21 * 10^{-35}\ T_{Kev}^{1/2}}$$

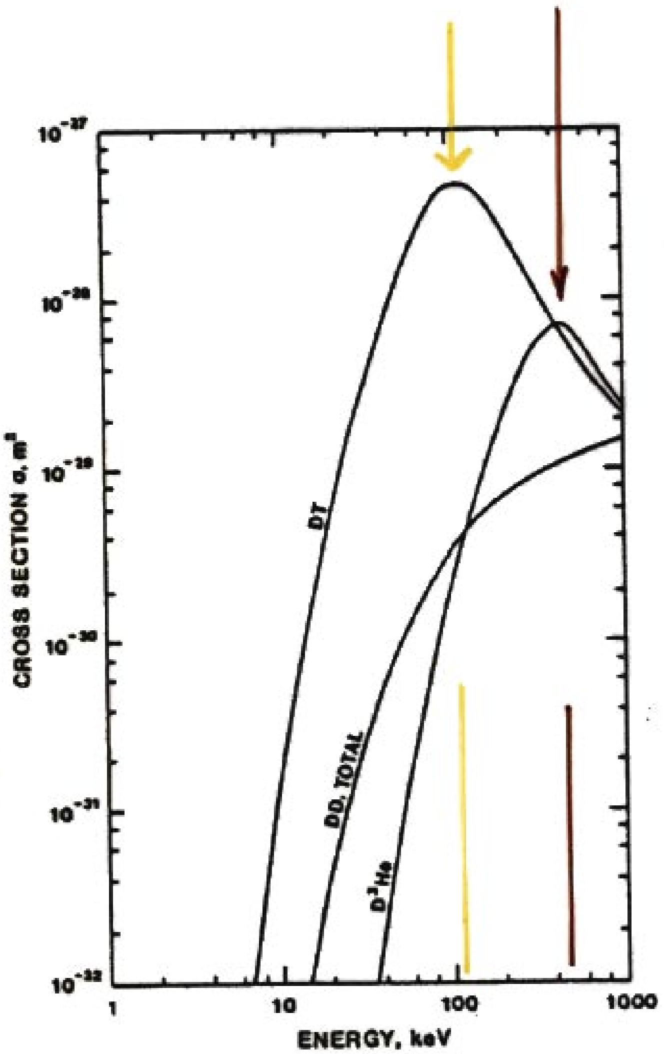
DT LAWSON DIAGRAM



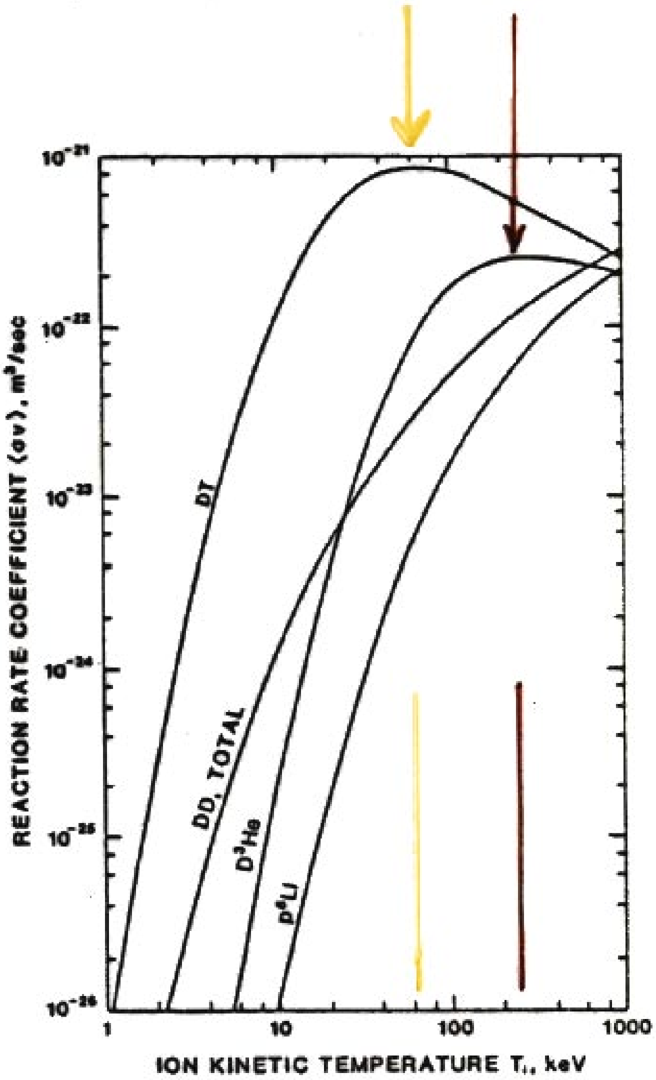
Lawson diagram for the DT fusion reaction. The parameters on the curves are the heating power fraction, Fhh_i. The required product of ion number density and particle containment time (Lawson parameter) is plotted as a function of the ion kinetic temperature.



Lawson diagram for the D^3-He fusion reaction. The required product of ion number density and particle containment time is plotted as a function of the ion kinetic temperature for several values of the heating power fraction, Fhh_i.



Fusion cross-sections
for selected reactants



Reaction rate coefficients
for selected reactants

D-T has the largest cross-section and highest reaction rate coefficient, at considerably lower energies as compared to other reactants, e.g. D-D or D-^{3}He.

Temperature Limits

if $\tau \gg \tau_{ie}$; then electron and ion temperatures will equilibrate, and $T_e = T_i = T$

The balance between P_c (fusion energy released in the form of charged particles) and P_{Rad} (The radiated power:

$$P_c = \underbrace{f_c}_{\text{fraction released in the form of charged particles}} P_{Fus} = P_{Rad}$$

$$= P_{BR} + P_{LR} + P_{cyc}$$

hence:

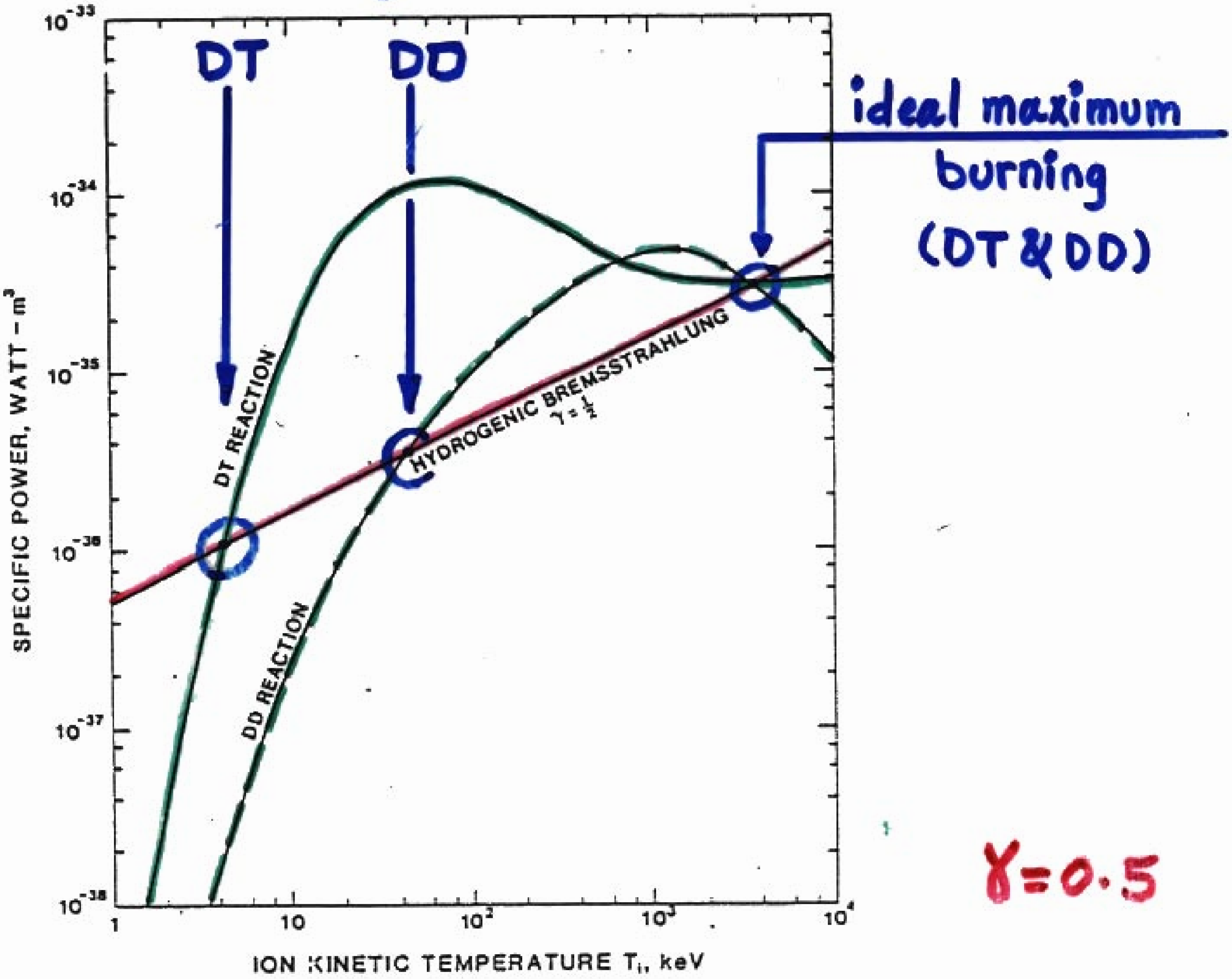
$$\approx P_{BR}$$

$$E_{12}\, g_{12} \langle \sigma v \rangle_{12} = 5.35 * 10^{-37}\, T^{1/2} [\gamma Z_1 + (1-\gamma) Z_2] * [\gamma Z_1^2 + (1-\gamma) Z_2^2]$$

M: doesn't appear, as this equilibrium is within the core

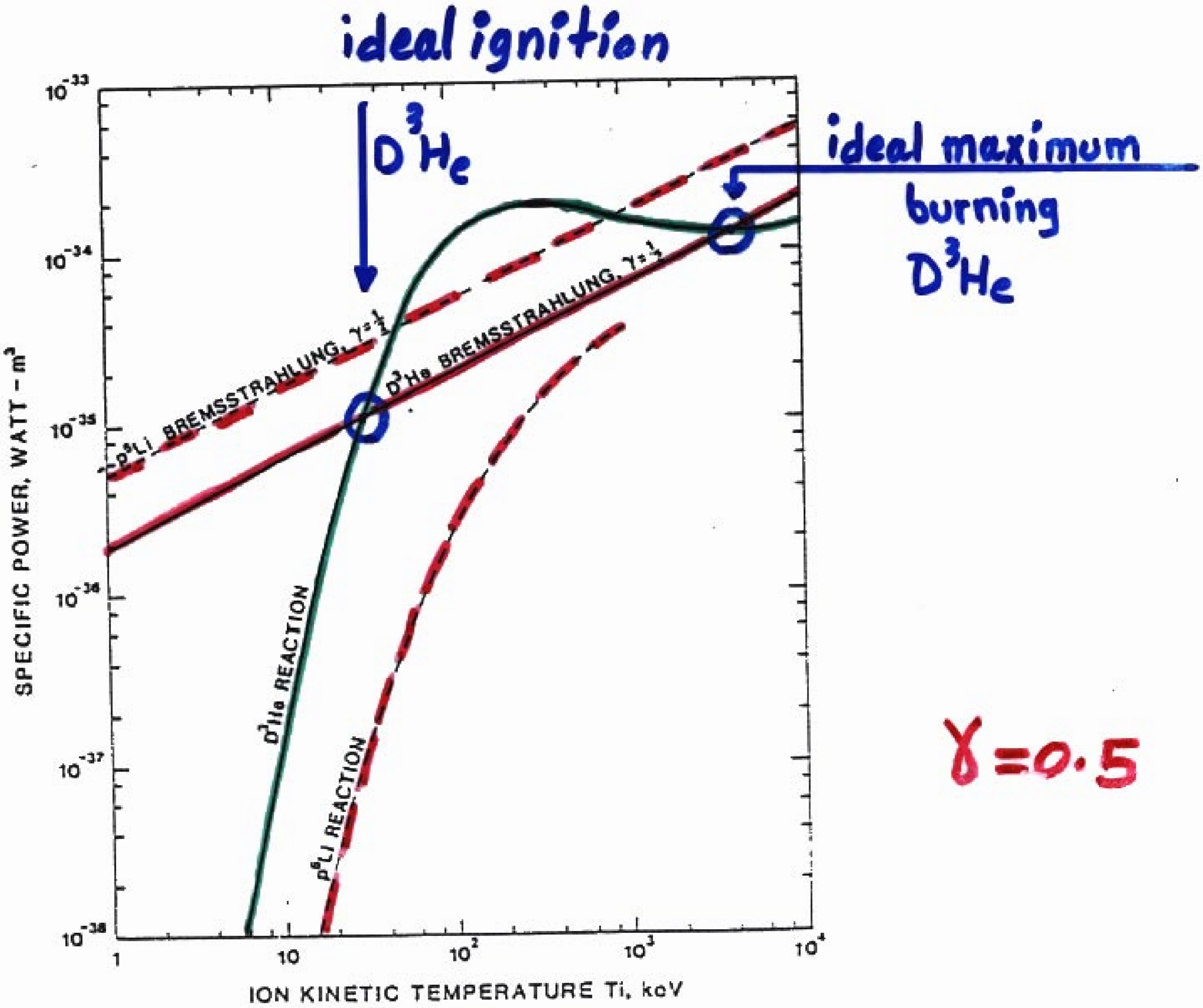
GRAPHICAL SOLUTION

ideal ignition



$\gamma = 0.5$

$$E_{12}\, g_{12} \langle \sigma v \rangle_{12} = 5.35 * 10^{-37}\, T^{1/2} [\gamma Z_1 + (1-\gamma) Z_2] * \cdots$$
$$\cdots [\gamma Z_1^2 + (1-\gamma) Z_2^2]$$



$\gamma = 0.5$

$$E_{12}\, g_{12} \langle \sigma v \rangle_{12} = 5.35 * 10^{-37}\, T^{1/2} \left[\gamma Z_1 + (1-\gamma) Z_2\right] * \cdots$$
$$\cdots \left[\gamma Z_1^2 + (1-\gamma) Z_2^2\right]$$

let us investigate the stability index β:

$$\beta = \frac{2\mu_0}{B^2} P = \frac{2\mu_0}{B^2} \sum_j n_j K T_j \leqslant 1$$

at equilibrium ; $T_e = T_i = T$

$$\beta = \frac{2\mu_0}{B^2} K T \underbrace{\sum_j n_j}_{\sum (n_e + n_i)}$$

$$n_e = \sum_j n_j Z_j = n_i [\gamma Z_1 + (1-\gamma) Z_2]$$

hence:

$$\sum_j n_j = n_i + n_i [\gamma Z_1 + (1-\gamma) Z_2]$$

$$= n_i [1 + \gamma Z_1 + (1-\gamma) Z_2]$$

$$= \frac{2\mu_0 K T}{B^2} n_i [1 + \gamma Z_1 + (1-\gamma) Z_2]$$

i.e. $$n_i = \frac{\beta B^2}{2\mu_0 K T [1 + \gamma Z_1 + (1-\gamma) Z_2]}$$

$$P_{FUS} = M g_{12} E_{12} n_i^2 \langle \sigma v \rangle_{12}$$

$$= M g_{12} E_{12} \langle \sigma v \rangle_{12} \frac{\beta^2 B^4}{4\mu_0^2 K^2 T^2 [\cdots]^2}$$

$$P_{FUS} = \frac{MB^4}{\mu_0^2}\left[\frac{g_{12}\, E_{12} \langle \sigma v \rangle_{12}}{K^2 T^2\, 4[\cdots]^2}\right]\beta^2$$

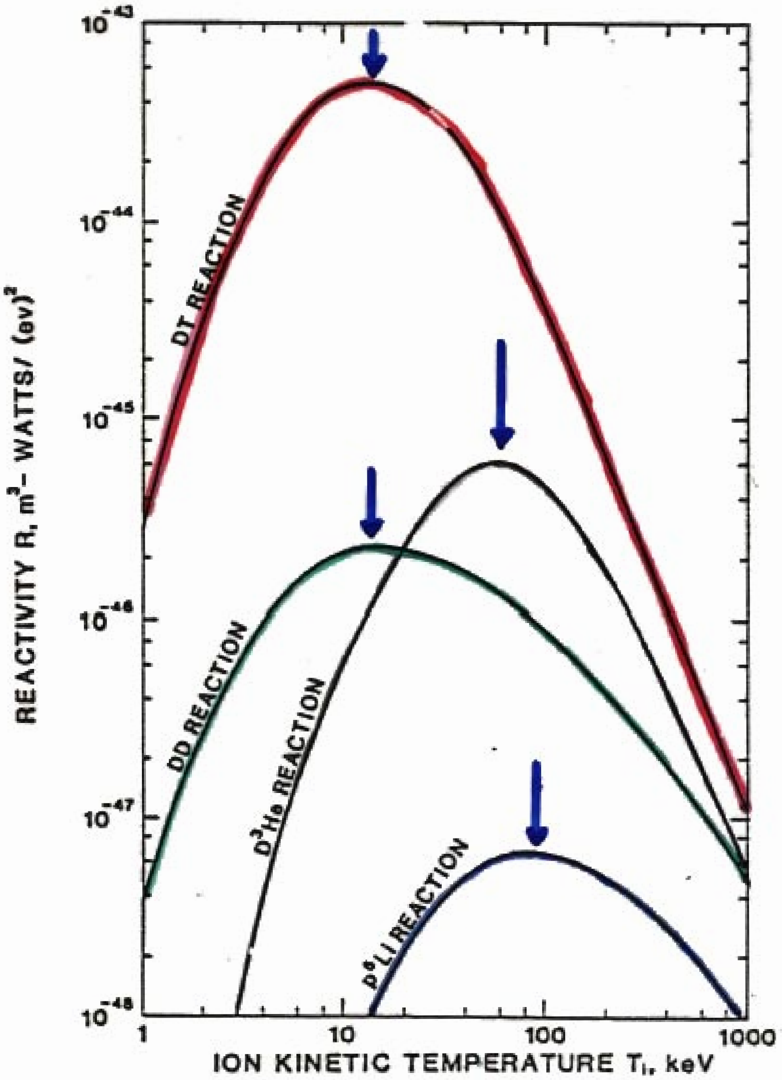
$$\frac{4[\cdots]^2}{g_{12}} = h_{12}$$

= fuel ion factor

$$= \frac{4}{g_{12}}\left[1+\gamma Z_1+(1-\gamma)Z_2\right]^2$$

$$= \frac{MB^4}{\mu_0^2}\underbrace{\left[\frac{E_{12}\langle \sigma v \rangle_{12}}{K^2 T^2 h_{12}}\right]}_{R_{12} \,\equiv\, \text{reactivity}}\beta^2$$

$$= \underbrace{\frac{MB^4}{\mu_0^2}}_{\text{depends upon the Confinment}}\ \underbrace{R_{12}}_{\text{function of the temp. } T,\, Z_1,\, Z_2}\beta^2$$



Optimum Operating Temperature

The reactivity of several fusion reactions as a function of kinetic temperature.

$$R_{12} = \frac{g_{12} E_{12} \langle \sigma v \rangle_{12}}{4K^2 T^2 \left[1 + \gamma Z_1 + (1-\gamma) Z_2\right]}$$